Sequences & Series - Past Questions & Solutions

November 2008

QUESTION 2

2.1 Consider the sequence: $\frac{1}{2};4;\frac{1}{4};7;\frac{1}{8};\ 10;\ldots$

2.1.1 If the pattern continues in the same way, write down the next TWO terms in the sequence. (2)

2.1.2 Calculate the sum of the first 50 terms of the sequence. (7)

2.2 Consider the sequence: 8 ; 18 ; 30 ; 44 ; …

2.2.1 Write down the next TWO terms of the sequence, if the pattern continues in the same way. (2)

2.2.2 Calculate the n^{th} term of the sequence. (6)

2.2.3 Which term of the sequence is 330? (4)

[21]

QUESTION 3

Given the geometric series: $8x^2+4x^3+2x^4+\ldots$

3.1 Determine the n^{th} term of the series. (1)

3.2 For what value(s) of x will the series converge? (3)

3.3 Calculate the sum of the series to infinity if $x=\frac{3}{2}$. (3)

[7]

2.2.1	60 ; 78	✓✓ answers (2)

2.2.2	8 18 30 44
	   10 12 14
	     2 2
	$2a = 2$
	$a = 1$
	$T_n = n^2 + bn + c$
	$8 = 1 + b + c$
	$7 = b + c \quad \ldots(\text{i})$
	$18 = 4 + 2b + c$
	$14 = 2b + c \quad \ldots(\text{ii})$
	(ii) – (i): $\quad 14 = 2b + c$
	$7 = b + c$
	$\therefore 7 = b$
	$c = 0$
	$T_n = n^2 + 7n$
	OR
		✓ $a = 1$
		✓ substitution
		✓ solving simultaneously
		✓ $b = 7$
		✓ $c = 0$
		✓ general term
		(6)

	$T_1 = 8$ $T_2 - T_1 = 10$ $T_3 - T_2 = 12$ $T_n - T_{n-1} =$ nth term of sequence with $a = 8$ and $d = 2$ Add both sides $T_n = 8 + 10 + 12 + \ldots +$ to 25 terms $T_n = \frac{n}{2}[16 + 2(n-1)]$ $T_n = n(n+7)$ **OR**	✓ $T_1 = 8$ ✓ $T_2 - T_1 = 10$ ✓ $T_3 - T_2 = 12$ ✓ Add both sides ✓ sequence ✓ substitution (6)

QUESTION 2

2.1.1	$\frac{1}{16}$; 13	✓✓ answers (2)
2.1.2	$\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots to\ 25\ terms\right)$ $\quad (4+7+10+13+\ldots to\ 25\ terms)$ $\frac{a(r^n-1)}{r-1} \quad = \frac{n}{2}[2a+(n-1)d]$ $=\frac{\frac{1}{2}\left(\left(\frac{1}{2}\right)^{25}-1\right)}{\frac{1}{2}-1} \quad = \frac{25}{2}[2(4)+24(3)]$ $=0{,}9999999 \quad = 1\ 000$ $S_{50}=1001{,}00$ **OR** S_{50} = 25 terms of 1st sequence + 25 terms of 2nd sequence $S_{50}=\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots to\ 25\ terms\right)+(4+7+10+13+\ldots to\ 25\ terms)$ $S_{50}=\frac{\frac{1}{2}\left(\left(\frac{1}{2}\right)^{25}-1\right)}{\frac{1}{2}-1}+\frac{25}{2}[2(4)+24(3)]$ $S_{50}=0{,}999999\ldots+1000$ $S_{50}=1001{,}00$	✓ formula for geometric series ✓ $\frac{\frac{1}{2}\left(\left(\frac{1}{2}\right)^{25}-1\right)}{\frac{1}{2}-1}$ ✓ answer for geometric series ✓ formula for linear series ✓ $\frac{25}{2}[2(4)+24(3)]$ ✓ 1000 ✓ answer (7) Note: If used 50 terms in each series: max 5/7 (answer then is 3876) Answer only: 6 / 7 Write out series and then correct answer: full marks Write out both series and not add them: 6 / 7

T_0 T_1 T_2 T_4 T_5

0 8 18 30 44

8 10 12 14

2 2 2

$T_0 = 0$

$a(0)^2 + b(0) + c = 0$

$c = 0$

constant second difference = 2

$a = 1$

$T_1 = 1 + b = 8$

$b = 7$

$T_n = n^2 + 7n$

$T_n = n(n+7)$

✓ finding T_0

✓ $c = 0$

✓ second difference = 2

✓ $a = 1$

✓ substitution

✓ $b = 7$

(6)

OR

$T_n = \frac{n-1}{2}\left[2(\textit{first first difference}) + (n-2)(\textit{second difference})\right] + T_1$

$T_n = \frac{n-1}{2}[2(10) + (n-2)(2)] + 8$

$T_n = 10(n-1) + (n-2)(n-1) + 8$

$T_n = 10n - 10 + n^2 - 3n + 2 + 8$

$T_n = n^2 + 7n$

✓ formula

✓✓ substitution

✓✓ simplification

✓ answer

(6)

OR

$T_n = (n-1)T_2 - (n-2)T_1 + 2nd\ difference\frac{(n-1)(n-2)}{2}$

$T_n = (n-1)(18) - (n-2)(8) + 2\frac{(n-1)(n-2)}{2}$

$T_n = 18n - 18 - 8n + 16 + n^2 - 3n + 2$

$T_n = n^2 + 7n$

✓✓ formula

✓✓ substitution

✓ simplification

✓ answer

(6)

OR

	$T_n = \dfrac{(n-2)(n-3)T_1 - 2(n-1)(n-3)T_2 + (n-2)(n-1)T_3}{2}$ $T_n = \dfrac{(n^2-5n+6)(8) - 2(n^2-4n+3)(18) + (n^2-3n+2)(30)}{2}$ $T_n = 4n^2 - 20n + 24 - 18n^2 + 72n - 54 + 15n^2 - 45n + 30$ $T_n = n^2 + 7n$ **OR** $T_1 = 8 = 1.8$ $T_2 = 18 = 2.9$ $T_3 = 30 = 3.10$ $T_4 = 44 = 4.11$ $T_n = n^2 + 7n$	✓ formula ✓✓ substitution ✓✓ simplification ✓ answer (6) ✓✓✓✓✓ observation ✓ answer (6) Note: By trial and error: 6 / 6 Answer only: 6 / 6
2.2.3	$n(n+7) = 330$ $n^2 + 7n - 330 = 0$ $(n+22)(n-15) = 0$ $n = -22$ or $n = 15$ $n = 15$ $\therefore 15^{th}$ term is 330. Note: 3 / 4 if did not reject $n = -22$ Answer only: 4 / 4 By trial and error and then write $n = 15$: 4 / 4 1 / 4 if just equate T_n that they found If linear T_n and valid answer : 2 / 4	✓ substitution ✓ standard form ✓ factorisation ✓ answer (4) **[21]**

QUESTION 3

3.1	$T_n = (8x^2)\left(\frac{x}{2}\right)^{n-1}$ **OR** $T_n = 8\left(\frac{1}{2}\right)^{n-1} .x^{n+1}$ **OR** $T_n = 16x\left(\frac{x}{2}\right)^{n}$ **OR** $T_n = 2^{4-n} x^{n+1}$	✓ answer (1)
3.2	$ratio = \frac{x}{2}$ $-1 < \frac{x}{2} < 1$ $-2 < x < 2$	✓ ratio ✓ inequality ✓ answer (3)
3.3	$S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{8x^2}{1-\frac{x}{2}}$ $S_\infty = \frac{8\left(\frac{3}{2}\right)^2}{1-\frac{1}{2}\left(\frac{3}{2}\right)}$ $S_\infty = 72$ **OR** $18 + \frac{27}{2} + \frac{81}{8} + \ldots$ $S_\infty = \frac{18}{1-\frac{3}{4}}$ $S_\infty = \frac{18}{\frac{1}{4}}$ $S_\infty = 72$	✓ substitution into formula for S_∞ ✓ substitution of $x = \frac{3}{2}$ ✓ answer (3) ✓ series ✓ substitution ✓ answer (3) Formula Incorrect: 0 / 3 **[7]**

QUESTION 2

2.1 Tebogo and Matthew's teacher has asked that they use their own rule to construct a sequence of numbers, starting with 5. The sequences that they have constructed are given below.

Matthew's sequence: 5 ; 9 ; 13 ; 17 ; 21 ; …
Tebogo's sequence: 5 ; 125 ; 3 125 ; 78 125 ; 1 953 125 ; …

Write down the n^{th} term (or the rule in terms of n) of:

2.1.1 Matthew's sequence (3)

2.1.2 Tebogo's sequence (2)

2.2 Nomsa generates a sequence which is both arithmetic and geometric. The first term is 1. She claims that there is only one such sequence. Is that correct? Show ALL your workings to justify your answer. (5)

[10]

QUESTION 3

Given: $\sum_{t=0}^{99}(3t-1)$

3.1 Write down the first THREE terms of the series. (1)

3.2 Calculate the sum of the series. (4)
[5]

QUESTION 4

The following sequence of numbers forms a quadratic sequence:

$-3\ ;\ -2\ ;\ -3\ ;\ -6\ ;\ -11\ ;\ \ldots$

4.1 The first differences of the above sequence also form a sequence. Determine an expression for the general term of the first differences. (3)

4.2 Calculate the first difference between the 35^{th} and 36^{th} terms of the quadratic sequence. (2)

4.3 Determine an expression for the n^{th} term of the quadratic sequence. (4)

4.4 Explain why the sequence of numbers will never contain a positive term. (2)
[11]

QUESTION 5

Data regarding the growth of a certain tree has shown that the tree grows to a height of 150 cm after one year. The data further reveals that during the next year, the height increases by 18 cm. In each successive year, the height increases by $\frac{8}{9}$ of the previous year's increase in height. The table below is a summary of the growth of the tree up to the end of the fourth year.

	First year	**Second year**	**Third year**	**Fourth year**
Tree height (cm)	150	168	184	$198\frac{2}{9}$
Growth (cm)		18	16	$14\frac{2}{9}$

5.1 Determine the increase in the height of the tree during the seventeenth year. (2)

5.2 Calculate the height of the tree after 10 years. (3)

5.3 Show that the tree will never reach a height of more than 312 cm. (3)
[8]

QUESTION 2

2.1.1	$T_n = 4n+1$ **OR** $T_n = 5+(n-1)(4)$ $\quad = 4n+1$ **NOTE**: If $T_n = 5+(n-1)(4)$ then full marks	✓✓✓ Answer only (3) ✓ $d=4$ ✓ substitution ✓ answer (3)
2.1.2	$T_n = 5(25)^{n-1}$	✓ $r=25$ ✓ answer (2)
2.2	The sequence is 1 ; $1+d$; $1+2d$; $1+3d$; …. (AP) and 1 ; r ; r^2 ; r^3 ; …. (GP) $\therefore 1+d=r$ and $d=r-1$ But $1+2d=r^2$ $1+2(r-1)=r^2$ $r^2-2r+1=0$ $(r-1)^2=0$ $r=1$ OR $r^2=1+2d$ $(1+d)^2=1+2d$ $1+2d+d^2=1+2d$ $d^2=0$ $d=0$ $r=1$ $\therefore d=0$ $\therefore$ the one and only such sequence is 1 ; 1; 1 ; ….. Nomsa is correct. **OR** $T_1=1$ Let the sequence be 1 ; a ; b ; … Geometric: $r=\frac{a}{1}=\frac{b}{a}$ $a^2=b$ Arithmetic: $d=a-1=b-a$ $2a-1=b$ $2a-1=a^2$ $0=a^2-2a+1$ $0=(a-1)^2$ $a=1$ $b=1$ Sequence is 1 ; 1 ; 1 ; ... Nomsa is correct If: Sequence is 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; ... Then $d=0$ $r=1$ Therefore only one sequence exists. Nomsa is correct Max 3 / 5 If the candidate only gives Sequence is 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; ... then 2 / 5 If $ar^{n-1}=a+(n-1)d$ only then 1 / 5	✓ $1+d=r$ ✓ $1+2d=r^2$ ✓ $r=1$ ✓ $d=0$ ✓reason (5) ✓ Setting up sequence ✓ $a^2=b$ ✓ $b=2a-1$ ✓ $a=1$ ✓$b=1$ (5) **[10]**

QUESTION 3

3.1	$-1+2+5+\ldots$ **OR** $-1\ ;\ 2\ ;\ 5$	✓ all three terms (1)
3.2	$S_n=-1+2+5+8+\ldots\text{to }100\text{ terms}$ $S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{100}=\frac{100}{2}[2(-1)+(100-1)(3)]$ $=50[-2+297]$ $=14\,750$ Answer only: 4 / 4 **OR** $S_n=-1+2+5+8+\ldots\text{to }100\text{ terms}$ $T_{100}=3(100)-4$ $=296$ $S_n=\frac{n}{2}[T_1+T_{100}]$ $S_{100}=\frac{100}{2}[-1+296]$ $=50[295]$ $=14\,750$ **NOTE**: If $S_n=-1+2+5+8+\ldots\text{to }99\text{ terms}$ $S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{99}=\frac{99}{2}[2(-1)+(99-1)(3)]$ $=\frac{99}{2}[-2+294]$ $=14454$ **Then 3 / 4** Apply consistent accuracy. This is the answer if series is $2+5+8+$ $S_n=2+5+8+\ldots\text{to }100\text{ terms}$ $S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{100}=\frac{100}{2}[2(2)+(100-1)(3)]$ $=50[4+297]$ $=15050$ **Then 4 / 4**	✓ formula ✓ $n=100$ ✓ substitution ✓ answer (4) **[5]**

QUESTION 4

4.1	The first differences are 1; -1; -3; -5; …. These form a linear pattern $T_n = 1 + (n-1)(-2)$ $\quad = 3 - 2n$ **OR** T_n **=** $-2n + 3$ **ANSWER ONLY:** Full marks	✓ pattern ✓ $d = -2$ ✓ answer (3)
4.2	Between the 35th and 36th terms of the quadratic sequence lies the 35th first difference 35th first difference $= 3 - 2(35)$ $= -67$ **OR** From the quadratic sequence: $P_{36} = -1158$ and $P_{35} = -1091$ 35th first difference $= -1158 - (-1091)$ $= -67$ If substitute and get $T_{35} = -2(35) + 3 = -67$ and $T_{36} = -2(36) + 3 = -69$, leading to the answer -2 then **1 / 2**	✓ substitution of 35 into $T_n = -2n + 3$ ✓ answer (2) ✓ $P_{36} = -1158$ and $P_{35} = -1091$ ✓ answer (2)
4.3	Second difference of terms is -2 . $P_n = an^2 + bn + c$ $a = -1$. $3a + b = 1$ $-3 + b = 1$ $b = 4$ $a + b + c = -3$ $-1 + 4 + c = -3$ $c = -6$ $P_n = -n^2 + 4n - 6$ If the general term has been worked out correctly in 4.2 and not redone in 4.3 but answer just written down then **4 / 4** **OR**	✓ $a = -1$ ✓ substitution ✓ $b = 4$ ✓ $c = -6$ (4)

4.3 contd	Second difference of terms is -2 . $P_n = an^2 + bn + c$ $a = -1$. $P_0 = -6 = c$ $P_n = -n^2 + bn - 6$ $-3 = -(1)^2 + (1)b - 6$ $b = 4$ $P_n = -n^2 + 4n - 6$ **OR** $P_n = \frac{n-1}{2}\left[2(\mathit{first\ first\ difference}) + (n-2)(\mathit{second\ difference})\right] + P_1$ $P_n = \frac{n-1}{2}[2(1) + (n-2)(-2)] - 3$ $P_n = n - 1 - (n-2)(n-1) - 3$ $P_n = n - 1 - n^2 + 3n - 2 - 3$ $P_n = -n^2 + 4n - 6$ **OR** $P_n = (n-1)P_2 - (n-2)P_1 + 2nd\ difference \frac{(n-1)(n-2)}{2}$ $P_n = (n-1)(-2) - (n-2)(-3) - 2\frac{(n-1)(n-2)}{2}$ $P_n = -2n + 2 + 3n - 6 - n^2 + 3n - 2$ $P_n = -n^2 + 4n - 6$ **OR** $P_n = \frac{(n-2)(n-3)T_1 - 2(n-1)(n-3)T_2 + (n-2)(n-1)T_3}{2}$ $P_n = \frac{(n^2 - 5n + 6)(-3) - 2(n^2 - 4n + 3)(-2) + (n^2 - 3n + 2)(-3)}{2}$ $P_n = \frac{-3n^2 + 15n - 18 + 4n^2 - 16n + 12 - 3n^2 + 9n - 6}{2}$ $P_n = -n^2 + 4n - 6$ **OR**	

4.3 contd	$P_2 - P_1 = T_1$ $P_3 - P_2 = T_2$ $P_4 - P_3 = T_3$ $P_n - P_{n-1} = T_{n-1}$ $P_n - P_1 = T_1 + T_2 + ... + T_{n-1}$ $P_n - P_1 = \frac{n-1}{2}[2(1) + (n-2)(-2)]$ $P_n - (-3) = (n-1)(3-n)$ $P_n = -n^2 + 4n - 6$	
4.4	Maximum value of T_n is $\frac{4(-1)(-6) - 4^2}{4(-1)} = -2$ The maximum value is negative and hence the sequence can not have any positive terms as the function is maximum valued	✓ max value – 2 ✓ explanation (2)
	OR $-n^2 + 4n - 6$ $= -(n-2)^2 + 4 - 6$ $= -(n-2)^2 - 2$ The function has a maximum-value of -2 and therefore the pattern will never have positive values.	✓ max value – 2 ✓ explanation (2)
	OR $T_n = -n^2 + 4n - 6$ $\frac{d}{dn}(T_n) = -2n + 4$ $0 = -2n + 4$ $n = 2$ $T_2 = -(2)^2 + 4(2) - 6$ $= -2$ The function has a maximum-value of -2 and therefore the pattern will never have positive values.	✓ max value – 2 ✓ explanation (2)
	OR As the sequence decreases from the second term onwards and the second term is negative, the sequence will never have a positive term.	✓✓ answer (2)
	OR $T_n = -n^2 + 4n - 6$ $\frac{d}{dn}(T_n) = -2n + 4$ $\frac{d}{dn}(T_n) < 0$ for $n > 2$ and $T_2 < 0$ so the sequence decreases and stays negative	✓✓ answer (2) **[11]**

QUESTION 5

5.1

First year: 150

Second year: 150 + 18 = 168

Third year: $168 + \frac{8}{9}(18) = 184$

Growth $= 18\left(\frac{8}{9}\right)^{n-2}$ after n years — ✓ general terms

17^{th} year growth is $18\left(\frac{8}{9}\right)^{17-2} = 3{,}08$ cm — ✓ answer (2)

	Yr 1	Yr 2	Yr 3	Yr 4	Yr 5	Yr 6	Yr 7	Yr 8	Yr 9
Ht	150	168	184	198,2	210,84	222,07	232,06	240,94	248,83
Inc		18	16	14,2	12,64	11,23	9,99	8,88	7,89
	Yr 10	**Yr 11**	**Yr 12**	**Yr 13**	**Yr 14**	**Yr 15**	**Yr 16**	**Yr 17**	
Ht	255,84	262,08	267,62	272,55	276,93	280,82	284,28	287,36	
Inc	7,01	6,24	5,54	4,93	4,38	3,89	3,46	3,08	

5.2

Height after 10 years

$$= 150 + \frac{18\left(1-\left(\frac{8}{9}\right)^{9}\right)}{1-\frac{8}{9}}$$

= 150 + 105,8768146 …

= 255,88 cm

OR

$$= 150 + \frac{18\left(\left(\frac{8}{9}\right)^{9}-1\right)}{\frac{8}{9}-1}$$

= 150 + 105,8768146 ...

= 255,88 cm

> **NOTE:**
> By writing out 9 terms and adding to 150 and answer correct, full marks
>
> Answer only: 2 / 3

✓ $n = 9$

✓ substitution into sum formula

✓ answer (3)

5.3

Max height = 150 + sum to infinity — ✓ statement

$$= 150 + \frac{18}{1-\frac{8}{9}}$$

— ✓ substitution into the sum to infinity formula

= 150 cm + 162 cm

= 312 cm — ✓ max height

The tree will never reach a height of more than 312 cm. (3)

[8]

NOTE:

If a candidate answers in 5.1 that the growth is $18\left(\frac{8}{9}\right)^{n-1} = 18\left(\frac{8}{9}\right)^{16} = 2{,}73$ cm then 1 / 2

The answer for 5.2 as continued accuracy uses $n = 10$,

Height after 10 years

$$= 150 + \frac{18\left(1-\left(\frac{8}{9}\right)^{10}\right)}{1-\frac{8}{9}} = 150 + 112{,}11 \ldots = 262{,}11 \text{ cm}$$

> **This is awarded 3/3 as consistent accuracy**

QUESTION 2

Consider the following sequence: 399 ; 360 ; 323 ; 288 ; 255 ; 224 ; …

2.1 Determine the n^{th} term T_n in terms of n. (6)

2.2 Determine which term (or terms) has a value of 0. (3)

2.3 Which term in the sequence will have the lowest value? (1)
[10]

QUESTION 3

3.1 Prove that: $a + ar + ar^2 + \ldots$ (to n terms) $= \dfrac{a(r^n - 1)}{r - 1}$, $r \neq 1$ (4)

3.2 Given the geometric series: $3 + 1 + \dfrac{1}{3} + \ldots$

Calculate the sum to infinity. (3)
[7]

QUESTION 4

Matli's annual salary is R120 000 and his expenses total R90 000. His salary increases by R12 000 each year while his expenses increase by R15 000 each year. Each year he saves the excess of his income.

4.1 Represent his total savings as a series. (4)

4.2 If Matli continues to manage his finances this way, after how many years will he have nothing left to save? (3)

4.3 Matli calculates that if his expenses increase by x rand every year (instead of R15 000 each year), he will spend as much as he earns in the 25^{th} year. Determine x. (2)
[9]

QUESTION 2

2.1	399 ; 360 ; 323 ; 288 ; 255 $-39 \quad -37 \quad -35 \quad -33$ $2 \quad 2 \quad 2$ Let $T_n = an^2 + bn + c$ Then $2a = 2$ $a = 1$ $T_1 = 399$: $a + b + c = 399$; $b + c = 398$ $T_2 = 360$: $4a + 2b + c = 360$; $2b + c = 356$ $b = -42$ $c = 440$ $T_n = n^2 - 42n + 440$	✓ 2^{nd} difference constant ✓ $a = 1$ ✓ $b + c = 398$ ✓ $2b + c = 356$ ✓ $b = -42$ ✓ $c = 440$ (6)
	OR $2a = 2$ $a = 1$ $T_2 - T_1 = -39$ $4a + 2b + c - a - b - c = -39$ $3a + b = -39$ $3 + b = -39$ $b = -42$ $a + b + c = 399$ $1 - 42 + c = 399$ $c = 440$ $T_n = n^2 - 42n + 440$	✓ 2^{nd} difference constant ✓ $a = 1$ ✓ $3a + b = -39$ ✓ $b = -42$ ✓ $a + b + c = 399$ ✓ $c = 440$ (6)
	OR $2a = 2$ $a = 1$ $399 - T_0 = -41$ $T_0 = 440.$ $But\ T_0 = c$ $\therefore$ c = 440 $T_n = n^2 + bn + 440$ $399 = 1^2 + b(1) + 440$ $399 - 441 = b$ $-42 = b$ $T_n = n^2 - 42n + 440$ **OR**	✓ 2^{nd} difference constant ✓ $a = 1$ ✓ $399 - T_0 = -41$ ✓ $c = 440$ ✓ $399 = 1^2 + b(1) + 440$ ✓ $b = -42$ (6)

	The sequence is 20^2-1 ; 19^2-1 ; 18^2-1 ; 17^2-1 ; ……. $T_1 = 20^2 = (20-0)^2 - 1$ $T_2 = 19^2 = (20-1)^2 - 1$ $T_3 = 18^2 = (20-2)^2 - 1$ $T_n = (20-(n-1))^2 - 1 = (21-n)^2 - 1$	✓✓rewriting terms as squares ✓✓✓ establishing that $T_n = (20-(n-1))^2$ ✓ $T_n = (21-n)^2$ (6)
2.2	$n^2 - 42n + 440 = 0$ $(n-22)(n-20) = 0$ $n = 22$ and $n = 20$ both terms 22 and 20 have values of 0. **OR** $(21-n)^2 - 1 = 0$ $21 - n = 1$ or -1 $n = 20$ or $n = 22$	✓ equation ✓✓ answers (3) ✓ equation ✓✓ answers (3)
2.3	$n = \frac{-(-42)}{2(1)}$ $n = 21$ At the 21st term, the lowest value is obtained. **OR** $2n - 42 = 0$ $2n = 42$ $n = 21$ $\therefore$ At the 21st term, the lowest value is obtained. **OR** $T_n = (21-n)^2 - 1 \therefore$ For $n = 21$, $T_n = (21-n)^2 - 1 = (21-21)^2 - 1 = -1$ For $n = 21$, the lowest value $(= -1)$ is obtained.	✓ answer (1) ✓ answer (1) ✓ answer (1) **[10]**

QUESTION 3

<table>
<tr><td>3.1</td><td>Let
$S_n = a + ar + ar^2 + ar^3 + \ldots\ldots + ar^{n-1}$ (1)
Then
$r \times S_n = r\left(a + ar + ar^2 + ar^3 + \ldots\ldots + ar^{n-1}\right)$
$= ar + ar + ar^2 + ar^3 + \ldots\ldots + ar^{n-1} + ar^n$ (2)
(2) – (1) gives:
$rS_n - S_n = ar^n - a$
$S_n(r-1) = a(r^n - 1)$
$S_n = \frac{a(r^n-1)}{(r-1)}$</td><td>✓ writing S_n as a series
✓ writing $r.S_n$ as a series
✓ subtracting
✓ removing common factors
(4)</td></tr>
<tr><td>3.2</td><td>$a = 3\,;\, r = \frac{1}{3}$
$S_\infty = \frac{a}{1-r}$
$= \frac{3}{1-\frac{1}{3}}$
$= \frac{9}{2}$</td><td>✓ $r = \frac{1}{3}$
✓ substitution
✓ answer
(3)
[7]</td></tr>
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QUESTION 4

<table>
<tr><td>4.1</td><td><table><tr><th>Term</th><th>Income</th><th>Expenses</th><th>Savings</th></tr><tr><td>1</td><td>120 000</td><td>90 000</td><td>30 000</td></tr><tr><td>2</td><td>132 000</td><td>105 000</td><td>27 000</td></tr><tr><td>3</td><td>144 000</td><td>120 000</td><td>24 000</td></tr></table>30 000 + 27 000 + 24 000 + …+ 0.</td><td>✓ 30 000
✓ 27 000
✓ 24 000
✓ series (4)</td></tr>
<tr><td>4.2</td><td>Savings = Income – Expenses
Income in year n = 120 000 + 12 000(n – 1)
Expenses in year n = 90 000 + 15 000(n – 1)
$120000 + 12000(n-1) = 90000 + 15000(n-1)$
$30000 + 12000n - 12000 = 15000n - 15000$
$33000 = 3000n$
$n = 11$
∴ After 11 years.
OR</td><td>✓✓ equating
✓ answer
(3)</td></tr>
</table>

	a = 30 000 d = –3000 $T_n = 30000 + (n-1)(-3000)$ $0 = 30000 - 3000n + 3000$ $3000n = 33000$ $\therefore n = 11$ $\therefore$ After 11 years	✓✓ equation ✓ answer (3)
4.3	$120000 + 12000(25-1) = 90000 + x(25-1)$ $x = 13250$	✓equating ✓ answer (2) **[9]**

November 2010

QUESTION 2

2.1 Evaluate: $\sum_{n=1}^{20} 3^{n-2}$ (4)

2.2 The following sequence forms a convergent geometric sequence: $5x \; ; \; x^2 \; ; \; \frac{x^3}{5} \; ; \; \ldots$

2.2.1 Determine the possible values of x. (3)

2.2.2 If $x = 2$, calculate S_∞. (2)

2.3 The following arithmetic sequence is given: 20 ; 23 ; 26 ; 29; … ; 101

2.3.1 How many terms are there in this sequence? (2)

2.3.2 The even numbers are removed from the sequence.
Calculate the sum of the terms of the remaining sequence. (6)

[17]

QUESTION 3

The sequence $4 \; ; \; 9 \; ; \; x \; ; \; 37; \ldots$ is a quadratic sequence.

3.1 Calculate x. (3)

3.2 Hence, or otherwise, determine the n^{th} term of the sequence. (4)

[7]

QUESTION 2

2.1	$\sum_{n=1}^{20} 3^{n-2}$ $= \frac{1}{3} + 1 + 3 + \ldots$ to 20 terms $= \frac{\frac{1}{3}\left(3^{20}-1\right)}{3-1}$; $r = 3;\ n = 20$ $= \frac{3^{20}-1}{6}$ $= 581130733{,}33$ OR $581130733\frac{1}{3}$ OR $581130733{,}3$ **Note:** The mark for $n = 20$ can be implied in the substitution to the formula **Note:** If leave only as $\frac{1}{3} + 1 + 3 + 9 + 27 + 81 + 243 + 729 + 2187 + 6561 + 19683$ $+ 59049 + 177147 + 531441 + 1594323 + 4782969$ $+ 14348907 + 43046721 + 129140163 + 387420489$ **only**, then 2 / 4 **Note:** The 20[th] term is **387 420 489** **Answer only:** 3 / 4 marks	✓ $a = \frac{1}{3}$ ✓ $r = 3$ ✓ $n = 20$ ✓ answer (4)
2.2.1	$5x\ ;\ x^2\ ;\ \frac{x^3}{5}\ ;\ \ldots$ $r = \frac{x}{5}$ $-1 < \frac{x}{5} < 1$ $-5 < x < 5$ **Answer can be written as** $x \in (-5\ ;\ 5)$ **Note:** If $-1 < x < 1$ 1 mark **Note:** If answer is $-5 \leq x \leq 5$ then 2 / 3	✓ $r = \frac{x}{5}$ or $\frac{x^2}{5x}$ ✓ $-1 < r < 1$ ✓ answer (3)
2.2.2	$r = \frac{2}{5}$ and $a = 10$ $S_\infty = \frac{10}{1-\frac{2}{5}}$ $= \frac{50}{3}$ or $16{,}67$	✓ $a = 10$ ✓ answer (2)

2.3.1	$T_n = 20 + 3(n-1)$ $101 = 20 + (n-1)3$ $84 = 3n$ $n = 28$ **OR** $T_n = 3n + 17$ $101 = 3n + 17$ $84 = 3n$ $n = 28$	**Note:** If $n = -\frac{17}{3}$ Then 1 / 2 marks **Answer only**: Full marks	✓ $101 = 20 + 3(n-1)$ or $101 = 3n + 17$ ✓ answer (2) ✓ substitution ✓ answer (2)
2.3.2	$23 + 29 + \ldots$ to 14 terms $= \frac{14}{2}[2(23) + (14-1)6]$ OR $\frac{14}{2}[23 + 101]$ $= 868$	**Note:** If "to 14 terms" is left out, do not penalise **Note:** If incorrect value for n, max 4 / 6 **Note:** If incorrect formula, max 2 / 6	✓ $23 + 29 + \ldots$ ✓ $a = 23$ ✓ $n = 14$ ✓ $d = 6$ or $l = 101$ ✓ substitution into correct formula ✓ answer (6)
	OR Even numbers = 20 ; 26 ;…; 98 $T_n = 6n + 14$ **OR** $T_n = 20 + (n-1)6$ $98 = 6n + 14$ **OR** $98 = 20 + (n-1)6$ $84 = 6n$ **OR** $84 = 6n$ $14 = n$ **OR** $14 = n$ $S_{remaining} = \frac{28}{2}[2(20) + (27)(3)] - \frac{14}{2}[2(20) + (13)(6)]$ $= 14(121) - 7(118)$ $= 1694 - 826$ $= 868$	**Note:** If the candidate only works out the even numbers i.e. 826, then 3 / 6 marks If only 1694 max 1 / 6 marks	**OR** ✓ $98 = 6n + 14$ or $98 = 20 + (n-1)$ ✓ $14 = n$ ✓substitution into correct formula ✓1694 ✓ 826 ✓ answer (6)
	OR Sequence is ~~20~~; 23; ~~26~~; 29; ~~32~~; 35; ~~38~~; 41; ~~44~~; 47; ~~50~~; 53; ~~56~~; 59; ~~62~~; 65; ~~68~~; 71; ~~74~~; 77; ~~80~~; 83; ~~86~~; 89; ~~92~~; 95; ~~98~~; 101 Sum of odd numbers = 23 + 29 + 35 + 41 + 47 + 53 + 59 + 65 + 71 + 77 + 83 + 89 + 95 + 101 = 868		Full marks (6) **[17]**

QUESTION 3

3.1	4 9 x 37 5 $x-9$ $37-x$ $x-14$ $37-x-(x-9)$ First difference : 5; $x-9$; $37-x$ Second difference : $x-14$; $-2x+46$ $x-14=46-2x$ $3x=60$ $x=20$ **Note:** Answer only: **Full Marks** **OR** $(x-9)+(x-14)=37-x$ $2x-23=37-x$ $3x=60$ $x=20$ **OR** $x+(x-9)+(x-14)=37$ $3x-23=37$ $3x=60$ $x=20$ **OR** $(x-9)-5=(37-x)-(x-9)$ $x-14=-2x+46$ $3x=60$ $x=20$	✓ first differences 5; $x-9$; $37-x$ ✓ seconds difference ✓ answer (3) ✓ equating ✓ manipulation ✓ answer (3) ✓ first differences 5; $x-9$; $37-x$ ✓ equating ✓ answer (3)
3.2	4 9 20 37 5 11 17 6 6 **Note:** If x is incorrect in 3.1 then **max 2 / 4 marks** $2a=6$ $a=3$ $T_n=3n^2+bn+c$ $3+b+c=4 \quad \ldots T_1$ $b+c=1$ $12+2b+c=9 \quad \ldots T_2$ $2b+c=-3$ $\therefore 9+b=5$ $b=-4$ and $c=4-(-1)=5$ $\therefore T_n=3n^2-4n+5$ **OR**	✓ $a=3$ ✓ $T_n=3n^2+bn+c$ ✓ $b=-4$ ✓ $c=5$ (4)

	$2a = 6$ $a = 3$ $T_0 = 5$ $c = 5$ $T_n = 3n^2 + bn + 5$ $4 = 3(1)^2 + b + 5$ $b = -4$ $T_n = 3n^2 - 4n + 5$ **OR** $2a = 6$ $a = 3$ $3a + b = 5$ $b = -4$ $a + b + c = 4$ $3 - 4 + c = 4$ $c = 5$ $T_n = 3n^2 - 4n + 5$	**OR** ✓ $a = 3$ ✓ $c = 5$ ✓ method ✓ $b = -4$ (4)
	OR $a + b + c = 4 \quad \ldots\text{i}$ $4a + 2b + c = 9 \quad \ldots\text{ii}$ $16a + 4b + c = 37 \quad \ldots\text{iii}$ $3a + b = 5$ $12a + 2b = 28$ $6a + b = 14$ $3a = 9$ $a = 3$ $b = -4$ $c = 5$ $T_n = 3n^2 - 4n + 5$ **OR** $T_n = 4 + (n-1)5 + \frac{6(n-1)(n-2)}{2}$ $= 4 + 5n - 5 + 3n^2 - 9n + 6$ $= 3n^2 - 4n + 5$	✓ $a = 3$ ✓ $c = 5$ ✓ method ✓ $b = -4$ (4) [7]

February 2011

QUESTION 2

The sequence 3 ; 9 ; 17 ; 27 ; … is a quadratic sequence.

2.1 Write down the next term. (1)

2.2 Determine an expression for the n^{th} term of the sequence. (4)

2.3 What is the value of the first term of the sequence that is greater than 269? (4)

[9]

QUESTION 3

3.1 The first two terms of an infinite geometric sequence are 8 and $\frac{8}{\sqrt{2}}$. Prove, without the use of a calculator, that the sum of the series to infinity is $16+8\sqrt{2}$. (4)

3.2 The following geometric series is given: $x = 5 + 15 + 45 + \ldots$ to 20 terms.

3.2.1 Write the series in sigma notation. (2)

3.2.2 Calculate the value of *x*. (3)

[9]

QUESTION 4

4.1 The sum to *n* terms of a sequence of numbers is given as: $S_n = \frac{n}{2}(5n+9)$

4.1.1 Calculate the sum to 23 terms of the sequence. (2)

4.1.2 Hence calculate the 23^{rd} term of the sequence. (3)

4.2 The first two terms of a geometric sequence and an arithmetic sequence are the same. The first term is 12. The sum of the first three terms of the geometric sequence is 3 more than the sum of the first three terms of the arithmetic sequence.

Determine TWO possible values for the common ratio, *r*, of the geometric sequence. (6)

[11]

QUESTION 2

2.1	39	✓ answer (1)

2.2	3 9 17 27
	   6 8 10
	     2 2
	Let $T_n = an^2 + bn + c$
	Then
	$2a = 2$
	$a = 1$
	$3a + b = 6$
	$3(1) + b = 6$
	$b = 3$
	$a + b + c = 3$
	$1 + 3 + c = 3$
	$c = -1$
	$T_n = n^2 + 3n - 1$
	
	OR
	
	$2a = 2$
	$a = 1$
	$c = 3 - 4 = -1$
	$T_n = n^2 + bn - 1$
	$3 = (1)^2 + b(1) - 1$ (using $T_1 = 3$)
	$b = 3$
	$T_n = n^2 + 3n - 1$
		✓ formula
		✓ $a = 1$
		✓ $b = 3$
		✓ $c = -1$
		(4)
		
		✓ $a = 1$
		✓ $c = -1$
		✓ formula
		✓ $b = 3$
		(4)

2.3	$n^2 + 3n - 1 > 269$ $n^2 + 3n - 270 > 0$ $(n + 18)(n - 15) > 0$ The first value of n is 16 The term is $16^2 + 3(16) - 1 = 303$	✓ $n^2 + 3n - 1 > 269$ ✓ factors ✓ $n = 16$ ✓ answer (4) **[9]**

QUESTION 3

3.1	$S_\infty = 8 + \frac{8}{\sqrt{2}} + \dots$ $r = \frac{1}{\sqrt{2}}$ and $S_\infty = \frac{a}{1-r}$ $= \frac{8}{1-\frac{1}{\sqrt{2}}}$ $= \frac{8\sqrt{2}}{\sqrt{2}-1}$ $= \frac{8\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$ $= 8\sqrt{2}\sqrt{2} + 8\sqrt{2}$ $= 16 + 8\sqrt{2}$ **OR** $S_\infty = 8 + \frac{8}{\sqrt{2}} + \dots$ $r = \frac{1}{\sqrt{2}}$ and $S_\infty = \frac{a}{1-r}$ $= \frac{8}{1-\frac{1}{\sqrt{2}}}$ $= \frac{8\left(1+\frac{1}{\sqrt{2}}\right)}{\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right)}$ $= \frac{8\left(1+\frac{1}{\sqrt{2}}\right)}{\frac{1}{2}}$ $= 16\left(1+\frac{1}{\sqrt{2}}\right)$ $= 16 + \frac{16\sqrt{2}}{2}$ $= 16 + 8\sqrt{2}$	✓ $r = \frac{1}{\sqrt{2}}$ ✓ substitution ✓ rationalisation ✓ simplification (4) ✓ $r = \frac{1}{\sqrt{2}}$ ✓ substitution ✓ rationalisation ✓ simplification (4)

3.2.1	$5 + 15 + 45 + \ldots + T_{20}$ $= \sum_{n=1}^{20} 5(3)^{n-1}$ **OR** $5 + 15 + 45 + \ldots + T_{20}$ $= 5\sum_{n=0}^{19} (3)^{n}$ **OR** $5 + 15 + 45 + \ldots + T_{20}$ $= 5\sum_{i=l}^{l+19} (3)^{i-l}$ for any $l \in Z$	✓ ✓answer (2) ✓ ✓answer (2) ✓ ✓answer (2)
3.2.2	$5 + 15 + 45 + \ldots + T_{20}$ $= \frac{5(3^{20} - 1)}{3 - 1}$ $= 8\ 716\ 961\ 000$	✓ formula ✓ substitution ✓ answer (3) **[9]**

QUESTION 4

4.1.1	$S_{23} = \frac{23}{2}(5(23) + 9)$ $= 1426$	✓ substitution ✓ answer (2)
4.1.2	$T_{23} = S_{23} - S_{22}$ $= 1426 - \frac{22}{2}(5(22) + 9)$ $= 1426 - 1309$ $= 117$	✓ statement ✓ $S_{22} = 1309$ ✓ answer (3)
4.2	Arithmetic Sequence: $12\ ;\ 12 + d\ ;\ 12 + 2d$ Geometric Sequence: $12\ ;\ 12r\ ;\ 12r^2$ $12 + d = 12r$ $d = 12r - 12$ $12 + 12r + 12r^2 = 12 + 12 + d + 12 + 2d + 3$ $12r^2 = 12 + 2(12r - 12) + 3$ $12r^2 = 12 + 24r - 24 + 3$ $12r^2 - 24r + 9 = 0$ $4r^2 - 8r + 3 = 0$ $(2r - 3)(2r - 1) = 0$ $r = \frac{3}{2}$ or $r = \frac{1}{2}$	✓ equation ✓ equation ✓ standard form ✓ factors ✓ answers (6)

	OR The 3^{rd} term of GP = 3 + 3^{rd} term of AP $12r^2 = 3 + 12 + 2d$ $12r^2 = 15 + 24r - 24$ $12r^2 - 24r + 9 = 0$ $4r^2 - 8r + 3 = 0$ $(2r - 3)(2r - 1) = 0$ $r = \frac{3}{2} \quad or \quad r = \frac{1}{2}$	✓ equation ✓ equation ✓ standard form ✓ factors ✓ answers **[11]**

November 2011

QUESTION 2

2.1 Given the sequence: $4\ ;\ x\ ;\ 32$

Determine the value(s) of x if the sequence is:

2.1.1 Arithmetic (2)

2.1.2 Geometric (3)

2.2 Determine the value of P if $P = \sum_{k=1}^{13} 3^{k-5}$ (4)

2.3 Prove that for any arithmetic sequence of which the first term is a and the constant difference is d, the sum to n terms can be expressed as $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$. (4)

[13]

QUESTION 3

The following sequence is a combination of an arithmetic and a geometric sequence:

$$3\ ;\ 3\ ;\ 9\ ;\ 6\ ;\ 15\ ;\ 12\ ;\ \ldots$$

3.1 Write down the next TWO terms. (2)

3.2 Calculate $T_{52} - T_{51}$. (5)

3.3 Prove that ALL the terms of this infinite sequence will be divisible by 3. (2)

[9]

QUESTION 4

A quadratic pattern has a second term equal to 1, a third term equal to – 6 and a fifth term equal to – 14.

4.1 Calculate the second difference of this quadratic pattern. (5)

4.2 Hence, or otherwise, calculate the first term of the pattern. (2)

[7]

QUESTION 2

2.1.1	$x-4=32-x$ $2x=36$ $x=18$ **Note:** If answer only: award 2/2 marks **OR** $a=4$ $a+2d=32$ $2d=28$ $d=14$ $x=14+4$ $x=18$ **Note:** If candidate writes $x-4 \quad 32-x$ only (i.e. omits equality) : 0/2 marks **OR** $x=\frac{4+32}{2}=18$	✓ $T_2-T_1=T_3-T_2$ ✓ answer (2) ✓ $a+2d=32$ and $a=4$ ✓ answer (2) ✓ substitutes correctly into arithmetic mean formula i.e. $\frac{4+32}{2}$ ✓ answers (2)
2.1.2	$\frac{x}{4}=\frac{32}{x}$ $x^2=128$ $x=\pm\sqrt{128}$ $x=\pm 8\sqrt{2}$ OR $x=\pm 11{,}31$ OR $x=\pm 2^{\frac{7}{2}}$ **Note:** If candidate writes $\frac{x}{4} \quad \frac{32}{x}$ only (i.e. omits equality) : 0/2 marks **OR** $a=4$ $r=\frac{x}{4}$ $ar^2=4\left(\frac{x}{4}\right)^2$ $32=4\left(\frac{x}{4}\right)^2$ $x^2=128$ $x=\pm\sqrt{128}$ $x=\pm 8\sqrt{2}$ or $x=\pm 11{,}31$ or $x=\pm 2^{\frac{7}{2}}$ **Note:** If only $x=\sqrt{128}$ then penalty 1 mark **OR** $x=\pm\sqrt{4\times 32}$ $x=\pm\sqrt{128}$ or $x=\pm 8\sqrt{2}$ or $x=\pm 11{,}31$ or $x=\pm 2^{\frac{7}{2}}$	✓ $\frac{T_2}{T_1}=\frac{T_3}{T_2}$ ✓ $x^2=128$ ✓ both answers (surd or decimal or exponential form) (3) ✓ $32=4\left(\frac{x}{4}\right)^2$ ✓ $x^2=128$ ✓ both answers (surd or decimal or exponential form) (3) ✓✓ substitutes correctly into geometric mean formula i.e. $\pm\sqrt{4\times 32}$ ✓ both answers (surd or decimal or exponential form) (3)

2.2	$P=\sum_{k=1}^{13} 3^{k-5}$ $=3^{1-5}+3^{2-5}+3^{3-5}+\ldots+3^{13-5}$ $=3^{-4}+3^{-3}+3^{-2}+\ldots+3^{8}$ $=\dfrac{3^{-4}(3^{13}-1)}{3-1}$ $=9841{,}49$ or $9841\frac{40}{81}$ or $\frac{797161}{81}$ **Note:** Correct answer only: 1/4 marks only	✓ $a=3^{-4}$ or $\frac{1}{81}$ ✓ $r=3$ ✓ subs into correct formula ✓ answer (4)
	OR $P=\sum_{k=1}^{13} 3^{k-5}$ $=3^{1-5}+3^{2-5}+3^{3-5}+\ldots+3^{13-5}$ $=3^{-4}+3^{-3}+3^{-2}+\ldots+3^{8}$ $=\frac{1}{81}+\frac{1}{27}+\frac{1}{9}+\ldots+6561$ $=9841{,}49$ or $9841\frac{40}{81}$ or $\frac{797161}{81}$ **Note:** If the candidate rounds off and gets 9841,46 (i.e. correct to one decimal place): DO NOT penalise for the rounding off.	✓✓ expand the sum ✓ 13 terms in expansion ✓ answer (4)
2.3	$S_n=a+[a+d]+[a+2d]+\ldots+[a+(n-2)d]+[a+(n-1)d]$ $S_n=[a+(n-1)d]+[a+(n-2)d]+\ldots+[a+d]+a$ $2S_n=[2a+(n-1)d]+[2a+(n-1)d]+\ldots+[2a+(n-1)d]+[2a+(n-1)d]$ $=n[2a+(n-1)d]$ $S_n=\frac{n}{2}[2a+(n-1)d]$	✓writing out S_n ✓ "reversing" S_n ✓expressing $2S_n$ ✓ grouping to get $2S_n=n[2a+(n-1)d]$ (4)
	OR $S_n=a+[a+d]+[a+2d]+\ldots+(T_n-d)+T_n$ $S_n=T_n+(T_n-d)+\ldots+[a+d]+a$ $2S_n=a+T_n+a+T_n+a+T_n+\ldots+a+T_n$ $=n[a+a+(n-1)d]$ $=[2a+(n-1)d]$ $S_n=\frac{n}{2}[2a+(n-1)d]$ **Note:** If a candidate uses a circular argument (eg $S_{n+1}=S_n+T_n$): max 1/4 marks (for writing out S_n) **Note:** If a candidate uses a specific linear sequence, then NO marks.	✓writing out S_n ✓ "reversing" S_n ✓expressing $2S_n$ ✓ grouping to get $2S_n=n[a+a+(n-1)d]$ (4) **[13]**

QUESTION 3

3.1	21; 24 **Note:** If candidate writes $T_8 = 21$ $T_7 = 24$: award 1/2 marks	✓ 21 ✓ 24 (2)
3.2	$T_{2k} = 3.2^{k-1}$ and so $T_{52} = 3.2^{26-1} = 100663296$ $T_{2k-1} = 3 + 6(k-1) = 6k - 3$ and so $T_{51} = 6(26) - 3 = 153$ $T_{52} - T_{51} = 100663296 - 153$ $= 100663143$ **Note:** If candidate writes out all 52 terms **and** gets correct answer: award 5/5 marks **Note:** If candidate used $k = 52$: max 2/5	✓ 3.2^{k-1} ✓ T_{52} ✓ $6k-3$ ✓ T_{51} ✓ answer (5)
	OR Consider sequence P: 3 ; 6 ; 12 … $P_n = 3.2^{n-1}$ $P_{26} = 3.2^{26-1} = 100663296$ Consider sequence Q: 3 ; 9 ; 15 … $Q_n = 6n - 3$ $Q_{26} = 6(26) - 3 = 153$ $T_{52} - T_{51} = P_{26} - Q_{26}$ $= 100663296 - 153$ $= 100663143$ **Note:** if candidate interchanges order i.e. does $T_{51} - T_{52}$ **:** max 4/5 marks **Note:** writes out all 52 terms and subtracts $T_{51} - T_{52}$ **:** max 4/5 marks	✓ $P_n = 3.2^{n-1}$ ✓ P_{26} ✓ $Q_n = 6n-3$ ✓ Q_{26} ✓ answer (5)

3.3	For all $n \in \mathbf{N}$, $n = 2k$ or $n = 2k-1$ for some $k \in \mathbf{N}$ If $n = 2k$: $T_n = T_{2k} = 3.2^{k-1}$ If $n = 2k-1$: $T_n = T_{2k-1}$ $= 6k-3$ $= 3(2k-1)$ In either case, T_n has a factor of 3, so is divisible by 3. **Note:** If a candidate only illustrates divisibility by 3 with a specific finite part of the sequence, not the general term: 0/2 marks **OR** $P_n = 3.2^{n-1}$ Which is a multiple of 3 $Q_n = 6n-3$ $= 3(2n-1)$ Which is also a multiple of 3 Since $T_n = Q_{2k-1}$ or $T_n = P_{2k}$ for all $n \in \mathbf{N}$, T_n is always divisible by 3 **OR** The odd terms are odd multiples of 3 and the even terms are 3 times a power of 2. This means that all the terms are multiples of 3 and are therefore divisible by 3.	✓ factors 3.2^{k-1} ✓ factors $3(2k-1)$ (2) ✓ factors 3.2^{n-1} ✓ factors $3(2n-1)$ (2) ✓ odd multiples of 3 ✓ 3 times a power of 2 (2) **[9]**

QUESTION 4

4.1 The second, third, fourth and fifth terms are 1 ; – 6 ; T_4 and – 14

First differences are: – 7 ; $T_4 + 6$; – 14 – T_4

So $T_4 + 6 + 7 = -14 - 2T_4 - 6$

$T_4 = -11$

$d = -11 + 6 + 7 = 2$ or $-14 + 22 - 6 = 2$

> **Note:** Answer only (i.e. $d = 2$) with no working: 3 marks

> **Note:** Candidate gives $T_4 = -11$ and $d = 2$ only: award 5/5 marks

✓ – 7
✓ $T_4 + 6$
✓ – 14 – T_4
✓ setting up equation $T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)$
✓ answer
(5)

OR

T_2		T_3		T_4		T_5
1		-6				-14
	-7		-7+d		-7+2d	
		d		d		

$$T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)$$
$$-15 = (-7 + 2d) + (-7 + d) + -7$$
$$-15 = -21 + 3d$$
$$6 = 3d$$
$$d = 2$$

> **Note:** Candidate uses trial and error **and** shows this: award 5/5 marks

✓ – 7
✓ $-7 + d$
✓ $-7 + 2d$
✓ setting up equation $T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)$
✓ answer
(5)

OR

$$4a + 2b + c = 1$$
$$9a + 3b + c = -6$$
$$5a + b = -7$$

$$25a + 5b + c = -14$$
$$16a + 2b = -8$$
$$10a + 2b = -14$$
$$6a = 6$$
$$a = 1$$
$$d = 2a = 2$$

✓ $4a + 2b + c = 1$
✓ $9a + 3b + c = -6$
✓ $25a + 5b + c = -14$
✓ solved simultaneously
✓ answer
(5)

OR

T_1		1		-6		T_4		-14
	$1 - T_1$		-7		T_4+6		-14 - T_4	
		T_1 - 8		T_4+13		-20-2 T_4		

$$T_4 + 13 = -20 - 2T_4$$
$$3T_4 = -33$$
$$T_4 = -11$$
$$d = -11 + 13$$
$$d = 2$$

✓ – 7
✓ $T_4 + 6$
✓ $-14 - T_4$
✓ setting up equation
✓ answer
(5)

<table>
<tr>
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<td>

OR

$$\begin{array}{ccccccccc} T_1 & & T_2 & & T_3 & & T_4 & & T_5 \\ x & & 1 & & -6 & & y & & -14 \\ & 1-x & & -7 & & y+6 & & -14-y & \\ & & -8+x & & y+13 & & -20-2y & & \end{array}$$

$$y+13=-20-2y$$
$$3y=-33$$
$$y=-11$$

Second difference = $y+13=-11+13=2$

</td>
<td>

✓ -7
✓ $y+6$
✓ $-14-y$

✓ setting up equation
✓ answer
(5)

</td>
</tr>
<tr>
<td>4.2</td>
<td>

$$\begin{array}{ccccc} T_1 & & 1 & & -6 \\ & -9 & & -7 & \\ & & 2 & & \end{array}$$

Note: Answer only: award 2/2 marks

Note:
If incorrect d in 4.1, 2/2 CA marks for $T_1=d+8$ (since $1-T_1=-7-d$)

$T_1=10$

OR

$$a=1$$
$$5a+b=-7$$
$$5(1)+b=-7$$
$$b=-12$$
$$a+b+c=1$$
$$4(1)+2(-12)+c=1$$
$$c=21$$
$$T_n=n^2-12n+21$$
$$T_1=(1)^2-12(1)+21$$
$$=10$$

OR

$T_4+13=-8+T_1$ $y+13=-8+x$
$-11+13=-8+T_1$ **OR** $-11+13=-8+x$
$T_1=10$ $x=10$

</td>
<td>

✓ method
✓ $T_1=10$
(2)

✓ method
✓ $T_1=10$
(2)

✓ method
✓ $T_1=10$
(2)
[7]

</td>
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</table>

QUESTION 2

Given the arithmetic series: $-7-3+1+\ldots+173$

2.1 How many terms are there in the series? (3)

2.2 Calculate the sum of the series. (3)

2.3 Write the series in sigma notation. (3)
[9]

QUESTION 3

3.1 Consider the geometric sequence: $4\ ;\ -2\ ;\ 1\ \ldots$

3.1.1 Determine the next term of the sequence. (2)

3.1.2 Determine n if the n^{th} term is $\frac{1}{64}$. (4)

3.1.3 Calculate the sum to infinity of the series $4-2+1\ \ldots$ (2)

3.2 If x is a REAL number, show that the following sequence can NOT be geometric:

$$1\ ;\ x+1\ ;\ x-3\ \ldots$$ (4)
[12]

QUESTION 4

An athlete runs along a straight road. His distance d from a fixed point P on the road is measured at different times, n, and has the form $d(n)=an^2+bn+c$. The distances are recorded in the table below.

Time (in seconds)	1	2	3	4	5	6
Distance (in metres)	17	10	5	2	r	s

4.1 Determine the values of r and s. (3)

4.2 Determine the values of a, b and c. (4)

4.3 How far is the athlete from P when $n=8$? (2)

4.4 Show that the athlete is moving towards P when $n<5$, and away from P when $n>5$. (4)
[13]

QUESTION 2

2.1	$T_n = a + (n-1)d$ $173 = -7 + (n-1)(4)$ $173 = -7 + 4n - 4$ $4n = 184$ $n = 46$ **OR** $T_n = 4n - 11$ $173 = 4n - 11$ $4n = 184$ $n = 46$	✓ $d = 4$ ✓ $T_n = -7 + 4(n-1)$ ✓ answer (3) ✓✓ $T_n = 4n - 11$ ✓ answer (3)
2.2	$S_n = \frac{n}{2}[a + l]$ $= \frac{46}{2}[-7 + 173]$ $= 23[166]$ $= 3\,818$ **OR** $S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{46}{2}[2(-7) + (45)(4)]$ $= 23[-14 + 180]$ $= 3\,818$	✓ subs of $n = 46$ ✓ subs of a and l into the correct formula ✓ answer (3) ✓ subs of $n = 46$ ✓ subs of a and d into the correct formula ✓ answer (3)
2.3	$\sum_{n=1}^{46}(4n - 11)$	✓ $n = 1$ ✓ top value = 46 ✓ $4n - 11$ (3) **[9]**

QUESTION 3

3.1.1	$r=-\frac{1}{2}$ $T_4=1\left(-\frac{1}{2}\right)$ $=-\frac{1}{2}$	✓ $r=-\frac{1}{2}$ ✓ answer (2)
3.1.2	$T_n=4\left(-\frac{1}{2}\right)^{n-1}$ $\frac{1}{64}=4\left(-\frac{1}{2}\right)^{n-1}$ $\frac{1}{256}=\left(-\frac{1}{2}\right)^{n-1}$ $\left(-\frac{1}{2}\right)^{8}=\left(-\frac{1}{2}\right)^{n-1}$ $8=n-1$ $n=9$ **OR** $T_n=-8\left(-\frac{1}{2}\right)^{n}$ $\frac{1}{64}=-8\left(-\frac{1}{2}\right)^{n}$ $\frac{1}{256}=\left(-\frac{1}{2}\right)^{n}$ $\left(-\frac{1}{2}\right)^{8}=\left(-\frac{1}{2}\right)^{n-1}$ $8=n-1$ $n=9$ **OR** $T_4=-\frac{1}{2}$ $T_5=\frac{1}{4}$ $T_6=-\frac{1}{8}$ $T_7=\frac{1}{16}$ $T_8=-\frac{1}{32}$ $T_9=\frac{1}{64}$ $n=9$	✓ $4\left(-\frac{1}{2}\right)^{n-1}$ ✓ substitution ✓ $\frac{1}{256}=\left(-\frac{1}{2}\right)^{n-1}$ ✓ answer (4) ✓ T_5 and T_6 ✓ T_7 ✓ T_8 ✓ answer (4)
3.1.3	$S_\infty=\frac{a}{1-r}$ $=\frac{4}{1-\left(-\frac{1}{2}\right)}$ $=\frac{8}{3}$	✓ substitution into correct formula ✓ answer (2)

3.2	For a geometric sequence: $\frac{x+1}{1}=\frac{x-3}{x+1}$ $x^2+2x+1=x-3$ $x^2+x+4=0$ $x=\frac{-1\pm\sqrt{1-4(1)(4)}}{2(1)}$ $x=\frac{-1\pm\sqrt{-15}}{2}$ **OR** $x^2+2x+1=x-3$ $x^2+x+4=0$ $\left(x+\frac{1}{2}\right)^2+\frac{15}{4}=0$ $\left(x+\frac{1}{2}\right)^2+\frac{15}{4}\geq\frac{15}{4}>0$ Solution is non-real. There is no x-value that makes the sequence geometric.	✓ $\frac{T_2}{T_1}=\frac{T_3}{T_2}$ ✓ standard form ✓ subs in quadratic formula ✓ non-real/no x-values (4)
	OR	
	For a geometric sequence: $\frac{x+1}{1}=\frac{x-3}{x+1}$ $x^2+2x+1=x-3$ $x^2+x+4=0$ $b^2-4ac=1-4(1)(4)$ $=-15$ Solution is non-real There is no x-value that makes the sequence geometric.	✓ $\frac{T_2}{T_1}=\frac{T_3}{T_2}$ ✓ standard form ✓ subs in discriminant ✓ non-real/no x-values (4) **[12]**

QUESTION 4

4.1	17 10 5 2 -7 -5 -3 2 2 $r=1$ $s=2$ **OR** **NOTE:** Candidates may do 4.2 first. $d(n)=n^2-10n+26$ $r=d(5)$ $=(5)^2-10(5)+26$ $=1$ $s=d(6)$ $=(6)^2-10(6)+26$ $=12$ **OR** $d(n)=(n-5)^2+1$ $r=d(5)$ $=(5-5)^2+1$ $=1$ $s=d(6)$ $=(6-5)^2+1$ $=2$	✓ complete pattern ✓ $r=1$ ✓ $s=2$ (3) ✓ $d(n)=n^2-10n+26$ ✓ $r=1$ ✓ $s=2$ (3) ✓ $d(n)=(n-5)^2+1$ ✓ $r=1$ ✓ $s=2$ (3)
4.2	$2a=2$ $a=1$ $3a+b=-7$ $\therefore 3(1)+b=-7$ $b=-10$ $\therefore a+b+c=17$ $1-10+c=17$ $c=26$ $\therefore d(n)=n^2-10n+26$ **OR**	✓ $a=1$ ✓ method ✓ $b=-10$ ✓ $c=26$ (4)

$$a+b+c=17$$
$$4a+2b+c=10$$
$$3a+b=-7$$
$$9a+3b=-21$$
$$9a+3b+c=5$$
$$-21+c=5$$
$$c=26$$
$$a+b=-9$$
$$4a+2b=-16$$
$$2a+2b=-18$$
$$2a=2$$
$$a=1$$
$$b=-10$$
$$d(n)=n^2-10n+26$$

OR

$$a+b+c=17$$
$$4a+2b+c=10$$
$$9a+3b+c=5$$
$$3a+b=-7$$
$$5a+b=-5$$
$$2a=2$$
$$a=1$$
$$3(1)+b=-7$$
$$b=-10$$
$$(1)-10+c=17$$
$$c=26$$
$$d(n)=n^2-10n+26$$

✓ method
✓ $a=1$
✓ $c=26$
✓ $b=-10$
(4)

OR

$$2a=2$$
$$a=1$$
$$c=26$$
$$d(n)=n^2+bn+26$$
$$17=(1)^2+b+26$$
$$b=-10$$
$$d(n)=n^2-10n+26$$

✓ method
✓ $a=1$
✓ $c=26$
✓ $b=-10$
(4)

OR

$$\mathrm{d(n)}=\frac{\mathrm{n}-1}{2}\left[2(\text{first difference})+(\mathrm{n}-2)(\text{second difference})\right]+\mathrm{d}(1)$$
$$d(n)=\frac{n-1}{2}\left[2(-7)+(n-2)(2)\right]+17$$
$$d(n)=\frac{n-1}{2}\left[-18-2n\right]+17$$
$$d(n)=(n-1)(-9-n)+17$$
$$d(n)=n^2-10n+26$$

✓ method
✓ $a=1$
✓ $c=26$
✓ $b=-10$
(4)

OR

	$d(n)=(n-1)d(2)-(n-2)d(1)+\text{second difference}\times\dfrac{(n-1)(n-2)}{2}$ $d(n)=(n-1)(10)-(n-2)(17)+\dfrac{2(n-1)(n-2)}{2}$ $d(n)=10n-10-17n+34+(n-1)(n-2)$ $d(n)=n^2-10n+26$ **OR** $d(n)=(n-5)^2+1$ $\quad=n^2-10n+26$ $a=1$ $b=-10$ $c=26$	✓ method ✓ $a=1$ ✓ $c=26$ ✓ $b=-10$ (4) ✓ method ✓ $a=1$ ✓ $c=26$ ✓ $b=-10$ (4)
4.3	$d(8)=(8)^2-10(8)+26$ $\quad=10\text{ m}$ **OR** By symmetry $d(8)$ $=d(5+3)$ $=d(5-3)$ $=d(2)$ $=10$ **OR** 17, 10, 5, 2, 1, 2, 5, 10 so $d(8)=10$	✓ subs $t=8$ ✓ answer (2) ✓ method ✓ answer (2) ✓ method ✓ answer (2)
4.4	Since the distance from P is decreasing for $n<5$ the athlete is moving towards P. Since the distance from P is increasing for $n>5$, the athlete is moving away from P. **OR** It is sufficient to show that d is decreasing when $n<5$ and increasing when $n>5$ $d(n)=n^2-10n+26$ $d'(n)=2n-10$ $d'(n)=2(n-5)$ For $n<5$, $\quad 2(n-5)<0$ $\quad d'(n)<0\ \therefore$ decreasing For $n>5$, $\quad 2(n-5)>0$ $\quad d'(n)>0\ \therefore$ increasing	✓✓ decreasing Moving towards ✓✓ increasing Moving away ✓ $d'(n)=2n-10$ ✓ $2(n-5)<0$ ✓ decreasing ✓ increasing (4) **[13]**

November 2012

QUESTION 2

2.1 $3x + 1$; $2x$; $3x - 7$ are the first three terms of an arithmetic sequence. Calculate the value of x. (2)

2.2 The first and second terms of an arithmetic sequence are 10 and 6 respectively.

2.2.1 Calculate the 11^{th} term of the sequence. (2)

2.2.2 The sum of the first n terms of this sequence is -560. Calculate n. (6)

[10]

QUESTION 3

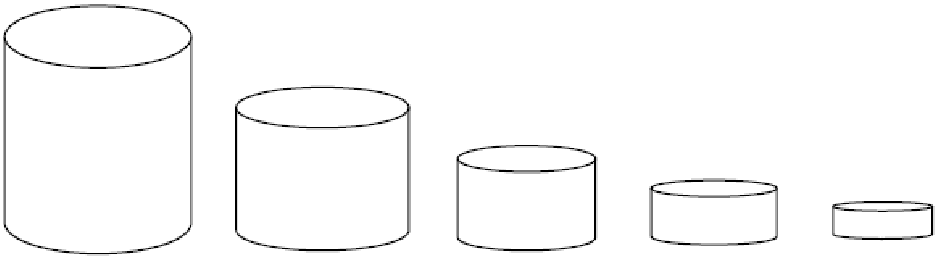
3.1 Given the geometric sequence: 27 ; 9 ; 3 …

3.1.1 Determine a formula for T_n, the n^{th} term of the sequence. (2)

3.1.2 Why does the sum to infinity for this sequence exist? (1)

3.1.3 Determine S_∞. (2)

3.2 Twenty water tanks are decreasing in size in such a way that the volume of each tank is $\frac{1}{2}$ the volume of the previous tank. The first tank is empty, but the other 19 tanks are full of water.



Would it be possible for the first water tank to hold all the water from the other 19 tanks? Motivate your answer. (4)

3.3 The n^{th} term of a sequence is given by $T_n = -2(n-5)^2 + 18$.

3.3.1 Write down the first THREE terms of the sequence. (3)

3.3.2 Which term of the sequence will have the greatest value? (1)

3.3.3 What is the second difference of this quadratic sequence? (2)

3.3.4 Determine ALL values of n for which the terms of the sequence will be less than -110. (6)

[21]

QUESTION 2

2.1	$T_2 - T_1 = T_3 - T_2$ $2x - (3x+1) = (3x-7) - 2x$ $2x - 3x - 1 = 3x - 7 - 2x$ $-x - 1 = x - 7$ $-2x = -6$ $x = 3$	✓ $T_2 - T_1 = T_3 - T_2$ or $2x - (3x+1) = (3x-7) - 2x$ ✓ answer (2)
	OR $T_2 = \frac{T_1 + T_3}{2}$ $2x = \frac{(3x+1)+(3x-7)}{2}$ $4x = 6x - 6$ $6 = 2x$ $x = 3$	✓ $T_2 = \frac{T_1 + T_3}{2}$ or $2x = \frac{(3x+1)+(3x-7)}{2}$ ✓ answer (2)
	OR $T_3 - T_1 = 2(T_2 - T_1)$ $(3x-7)-(3x+1) = 2(2x-(3x+1))$ $-8 = -2x - 2$ $2x = 6$ $x = 3$	✓ $T_3 - T_1 = 2(T_2 - T_1)$ or $(3x-7)-(3x+1) = 2(2x-(3x+1))$ ✓ answer (2)
2.2.1	$T_n = a + (n-1)d$ $T_{11} = 10 + (11-1)(-4)$ $= -30$	✓ $d = -4$ ✓ answer (2)
	OR 10; 6; 2; –2; –6; –10; –14; –18; –22; –26; –30 … $\therefore T_{11} = -30$	✓ expands sequence ✓ answer (2)

2.2.2	$S_n=\frac{n}{2}[2a+(n-1)d]$	✓correct formula ✓substitution of a and d
	$-560=\frac{n}{2}[2(10)+(n-1)(-4)]$	✓subs $S_n=-560$
	$-1120=-4n^2+24n$ $4n^2-24n-1120=0$ $n^2-6n-280=0$	✓ $-4n^2+24n+1120=0$ or $4n^2-24n-1120=0$ or $n^2-6n-280=0$
	$(n-20)(n+14)=0$	✓factors
	$n=20$ or -14 $\therefore n=20$ only	✓ selects $n=20$ only (6)
	Note: if candidate substitutes into **incorrect formula**, award 0/6	
	OR	
	$S_n=\frac{n}{2}[2a+(n-1)d]$	✓ correct formula
	$-560=\frac{n}{2}[2(10)+(n-1)(-4)]$ **Note:** if candidate writes **answer only**, award 1/6 marks	✓substitution of a and d ✓subs $S_n=-560$
	$-1120=-4n^2+24n$ $4n^2-24n-1120=0$ $n^2-6n-280=0$	✓ $4n^2-24n-1120=0$ or $-4n^2+24n+1120=0$ or $n^2-6n-280=0$
	$n=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(-280)}}{2(1)}$	✓subs into correct formula
	$n=20$ or -14 $\therefore n=20$ only	✓ selects $n=20$ only (6)
	OR	
	$S_n=\frac{n}{2}[2a+(n-1)d]$	✓ correct formula
	$-560=\frac{n}{2}[2(10)+(n-1)(-4)]$	✓substitution of a and d ✓subs $S_n=-560$
	$-560=\frac{20n}{2}-\frac{4n^2}{2}+\frac{4n}{2}$ $2n^2-12n-560=0$ $n^2-6n-280=0$	✓ $2n^2-12n-560=0$ or $-2n^2+12n+560=0$ or $n^2-6n-280=0$
	$(n-20)(n+14)=0$	✓factors
	$n=20$ or -14 $\therefore n=20$ only	✓ selects $n=20$ only (6)

OR

$S_{11} = -110$

n	12	13	14	15	16	17	18	19	20
T_n	–34	–38	–42	–46	–50	–54	–58	–62	–66
S_n	–144	–182	–224	–270	–320	–374	–432	–494	–560

$\therefore n = 20$

✓ $S_{11} = -110$

✓ sequence expanded

✓✓ series calculated

✓✓ answer

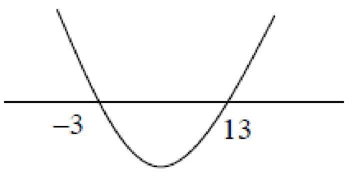
(6)

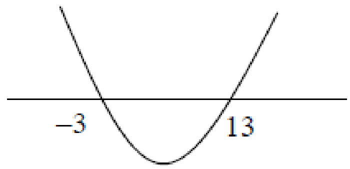
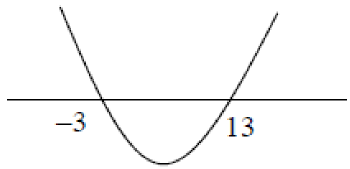
[10]

QUESTION 3

3.1.1	$T_n = ar^{n-1}$ $= 27\left(\frac{1}{3}\right)^{n-1}$ **Note:** The final answer can also be written as 3^{4-n} **or** $\left(\frac{1}{3}\right)^{n-4}$	✓ $a = 27$ and $r = \frac{1}{3}$ ✓ substitute into correct formula (2)
3.1.2	$-1 < r < 1$ or $\lvert r \rvert < 1$ **Note:** If candidate concludes series is not convergent, award 0 marks.	✓ answer (1)
	OR The common ratio (r) is $\frac{1}{3}$ which is between -1 and 1.	✓ answer (1)
	OR $-1 < \frac{1}{3} < 1$	✓ answer (1)
3.1.3	$S_\infty = \frac{a}{1-r}$ $= \frac{27}{1-\frac{1}{3}}$ $= \frac{81}{2}$ or 40,5 or 41 **Note:** If $r > 1$ or $r < -1$ is substituted then 0/2 marks.	✓ substitution ✓ answer (2)

3.2	Let V be the volume of the first tank. $\frac{V}{2};\frac{V}{4};\frac{V}{8}......$ $S_{19}=\frac{\frac{V}{2}\left[1-\left(\frac{1}{2}\right)^{19}\right]}{1-\frac{1}{2}}$ $=\frac{524287}{524288}V$ $=0,9999980927\ V$ $<V$ Yes, the water will fill the first tank without spilling over. **Note:** If candidate lets the volume of the first tank be a specific value (instead of a variable) and his/her argument follows correctly, award 4/4 marks **Note:** If candidate answers 'Yes' only with no justification: 1/4 marks	✓ $\frac{V}{2}$ ✓ substitute into correct formula ✓ answer ✓ conclusion (4)
	OR Let V be the volume of the first tank. $\frac{V}{2};\frac{V}{4};\frac{V}{8}......$ $S_{19}=\frac{\frac{V}{2}\left[1-\left(\frac{1}{2}\right)^{19}\right]}{1-\frac{1}{2}}$ $=V\left[1-\left(\frac{1}{2}\right)^{19}\right]$ $<V\cdot 1$ $=V$ Yes, the water will fill the first tank without spilling over.	✓ $\frac{V}{2}$ ✓ substitute into correct formula ✓ observes that $\left[1-\left(\frac{1}{2}\right)^{19}\right]<1$ ✓ conclusion (4)
	OR Let V be the volume of the first tank. $\frac{V}{2};\frac{V}{4};\frac{V}{8}......$ $S_{\infty}=\frac{\frac{V}{2}}{1-\frac{1}{2}}$ $=V$ Since the first tank will hold the water from infinitely many tanks without spilling over, certainly: Yes, the first tank will hold the water from the other 19 tanks without spilling over.	✓ $\frac{V}{2}$ ✓ substitute into correct formula ✓✓ correct argument (4)

	OR If the tanks are emptied one by one, starting from the second, each tank **will fill only half the remaining space**, so the first tank can hold all the water from the other 19 tanks.	✓ Yes (explicit or understood from the argument.) ✓✓✓ argument (4)
3.3.1	$T_n = -2(n-5)^2 + 18$ Term 1 = – 14 Term 2 = 0 Term 3 = 10	✓ – 14 ✓ 0 ✓ 10 (3)
3.3.2	Term 5 **OR** $n = 5$ **OR** T_5	✓ answer (1)
3.3.3	Second difference = $2a$ Second difference = $2(-2)$ Second difference = – 4 **OR** -14 0 10 14 10 – 4 Second difference = – 4	✓ subs – 2 into $2a$ ✓ answer (2) ✓ first differences ✓second difference (2)
3.3.4	$-2(n-5)^2 + 18 < -110$ $-2(n-5)^2 + 128 < 0$ $-2n^2 + 20n - 50 + 128 < 0$ $-2n^2 + 20n + 78 < 0$ $n^2 - 10n - 39 > 0$ $(n-13)(n+3) > 0$ + 0 − 0 + -3 13 $n < -3$ or $n > 13$ **Note:** Answer only award 2/6 marks  $n \geq 14$; $n \in \mathrm{N}$ **OR** $n > 13$; $n \in \mathrm{N}$ **OR**	✓ $T_n < -110$ ✓standard form ✓factors ✓ critical values ✓ inequalities ✓ $n > 13$ (accept: $n \geq 14$) (6)

	$-2(n-5)^2+18<-110$	✓ $T_n<-110$
	$-2(n-5)^2+128<0$	
	$(n-5)^2-64>0$	✓ $(n-5)^2-64>0$
	$[(n-5)-8][(n-5)+8]>0$	✓factors
	$(n-13)(n+3)>0$	
	$+\quad 0\quad -\quad 0\quad +$ $-3 \qquad\qquad 13$ 	✓ critical values ✓ inequalities ✓ $n>13$ (accept: $n\geq 14$)
	$n<-3$ or $n>13$	(6)
	$n\geq 14$; $n\in N$ **OR** $n>13$; $n\in N$	
	OR	
	$-2(n-5)^2+18<-110$	✓ $T_n<-110$
	$-2(n-5)^2<-128$	✓ $2(n-5)^2>128$
	$(n-5)^2>64$	✓ 8 and -8
	$n-5<-8$ or $n-5>8$	✓ $n-5>8$ ✓ $n-5<-8$
	$n<-3$ or $n>13$	✓ $n>13$ (accept: $n\geq 14$)
	$n\geq 14$; $n\in N$ **OR** $n>13$; $n\in N$	(6)
	OR	
	$T_n=-2(n-5)^2+18$	✓ $T_n<-110$
	$T_n=-2n^2+20n-32$	
	$-2n^2+20n-32<-110$	
	$-2n^2+20n-78<0$	✓standard form
	$n^2-10n-39>0$	
	$(n-13)(n+3)>0$	✓factors ✓ critical values
	$+\quad 0\quad -\quad 0\quad +$ $-3 \qquad\qquad 13$ 	
	$n<-3$ or $n>13$	✓ inequalities ✓ $n>13$
	$n\geq 14$; $n\in N$ **OR** $n>13$; $n\in N$	(accept: $n\geq 14$) (6)
	OR	
	-14 ; 0 ; 10 ; 16 ; 18 ; 16 ; 10 ; 0 ; -14 ; -32 ; -54 ; -80 ; -110	✓✓✓✓ expansion
	$n\geq 14$; $n\in N$	✓✓ conclusion of $n\geq 14$ (accept $n>13$) (6) **[21]**

QUESTION 2

2.1 Given the geometric series: $256 + p + 64 - 32 + \ldots$

2.1.1 Determine the value of p. (3)

2.1.2 Calculate the sum of the first 8 terms of the series. (3)

2.1.3 Why does the sum to infinity for this series exist? (1)

2.1.4 Calculate S_∞ (3)

2.2 Consider the arithmetic sequence: $-8 ; -2 ; 4 ; 10 ; \ldots$

2.2.1 Write down the next term of the sequence. (1)

2.2.2 If the n^{th} term of the sequence is 148, determine the value of n. (3)

2.2.3 Calculate the smallest value of n for which the sum of the first n terms of the sequence will be greater than 10 140. (5)

2.3 Calculate $\sum_{k=1}^{30}(3k+5)$ (3)

[22]

QUESTION 3

Consider the sequence: $3 ; 9 ; 27 ; \ldots$

Jacob says that the fourth term of the sequence is 81.
Vusi disagrees and says that the fourth term of the sequence is 57.

3.1 Explain why Jacob and Vusi could both be correct. (2)

3.2 Jacob and Vusi continue with their number patterns.

Determine a formula for the n^{th} term of:

3.2.1 Jacob's sequence (1)

3.2.2 Vusi's sequence (4)

[7]

QUESTION 2

2.1.1	$r=-\frac{32}{64}=-\frac{1}{2}$ $p=256\left(-\frac{1}{2}\right)$ $p=-128$	✓ $-\frac{1}{2}$ ✓ substitution ✓ answer (3)
	OR	
	$\frac{p}{256}=\frac{64}{p}$ $p^2=16384$ $p=\pm 128$ $p=-128$	✓ $\frac{p}{256}=\frac{64}{p}$ ✓ $p=\pm 128$ ✓ answer (3)
	OR	
	$\frac{p}{256}=\frac{-32}{64}$ $64p=8192$ $p=-128$	✓ $\frac{p}{256}=\frac{-32}{64}$ ✓ simplification ✓ answer (3)
	OR	
	$\frac{1}{r}=\frac{64}{-32}=-2$ $p=-2\times 64$ $p=-128$	✓ $\frac{1}{r}=\frac{64}{-32}=-2$ ✓ simplification ✓ answer (3)
2.1.2	$S_n=\frac{a\left[1-r^n\right]}{1-r}$ $S_8=\frac{256\left[1-\left(-\frac{1}{2}\right)^8\right]}{1+\frac{1}{2}}$ $=\frac{512}{3}\left(\frac{255}{256}\right)$ $=170$	✓ formula ✓ substitution ✓ answer (3)
	OR	

	$S_n = \dfrac{a\left[1-r^n\right]}{1-r}$ $S_8 = \dfrac{2^8\left[1-\left(-\frac{1}{2}\right)^8\right]}{1+\frac{1}{2}}$ $= \dfrac{2^9}{3}\left(\dfrac{255}{2^8}\right)$ $= 170$	✓ formula ✓ substitution ✓ answer (3)
2.1.3	$-1 < r < 1$ **OR** The common ratio is $-\dfrac{1}{2}$ which is between -1 and 1. **OR** $-1 < -\dfrac{1}{2} < 1$	✓ answer (1) ✓ answer (1) ✓ answer (1)
2.1.4	$S_\infty = \dfrac{a}{1-r}$ $= \dfrac{256}{1-\left(-\frac{1}{2}\right)}$ $= \dfrac{512}{3}$ $= 170{,}67$	✓ formula ✓ substitution ✓ answer (3)

2.2.1	16	✓ answer (1)
2.2.2	$T_n = -8 + 6(n-1)$ $148 = 6n - 14$ $6n = 162$ $n = 27$	✓ substitution into equation ✓ $T_n = 148$ ✓ answer (3)
2.2.3	$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $\frac{n}{2}\left[2(-8) + (n-1)(6)\right] > 10\,140$ $3n^2 - 11n > 10\,140$ $3n^2 - 11n - 10\,140 > 0$ $(3n + 169)(n - 60) > 0$ When $n = 60$, $S_n = 10\,140$ Smallest $n = 61$	✓ $\frac{n}{2}\left[2(-8) + (n-1)(6)\right]$ ✓ $3n^2 - 11n > 10\,140$ ✓ factors ✓ $n = 60$ ✓ answer (5)
2.3	$\sum_{k=1}^{30}(3k+5)$ $a = 8 \quad n = 30 \quad d = 3$ $\sum_{k=1}^{30}(3k+5) = \frac{30}{2}\left[2(8) + 29(3)\right]$ $= 15(103)$ $= 1545$	✓ $n = 30$ ✓ substitution into correct formula ✓ answer (3) **[22]**

QUESTION 3

3.1	Jacob calculated that the sequence is geometric or exponential. Vusi calculated that the sequence is quadratic. **OR** Jacob has multiplied each term by 3 to get the next term. Vusi sees it as a sequence with a constant second difference. **OR** Jacob calculated that the sequence is geometric or exponential. Vusi calculated that the sequence can be seen as a combination of exponential and cubic sequences.	✓Jacob (geometric/exponential) ✓ Vusi (quadratic) (2) ✓ Jacob (multiplied each term by 3) ✓ Vusi (constant second difference) (2) ✓Jacob (geometric/exponential) ✓ Vusi (exponential and cubic combined) (2)
3.2.1	$T_n = 3^n$ **OR** $T_n = 3.3^{n-1}$	✓answer (1) ✓answer (1)
3.2.2	3 9 27 57 6 18 30 12 12 $2a = 12$ $3a + b = 6$ $a + b + c = 3$ $a = 6$ $18 + b = 6$ $6 - 12 + c = 3$ $b = -12$ $c = 9$ $T_n = 6n^2 - 12n + 9$ **OR** $2a = 12$ $a = 6$ $T_0 = c = 9$ $T_n = an^2 + bn + 9$ $3 = 6(1)^2 + b(1) + 9$ $b = -12$ $T_n = 6n^2 - 12n + 9$ **OR**	✓ $a = 6$ ✓ method ✓ $b = -12$ ✓ $c = 9$ (4) ✓ $a = 6$ ✓ $c = 9$ ✓ method ✓ $b = -12$ (4)

	$2a = 12$ $a = 6$ $T_n = 6n^2 + bn + c$ $T_1 = 3 = 6(1)^2 + b(1) + c$ i.e. $3 = 6 + b + c$ $T_2 = 9 = 6(2)^2 + b(2) + c$ i.e. $\underline{9 = 24 + 2b + c}$ $6 = 18 + b$ $b = -12$ $c = 9$ $T_n = 6n^2 - 12n + 9$ **OR** $T_n = 3^n + k(n-1)(n-2)(n-3)$ $57 = 3^4 + k(3)(2)(1)$ $6k = -24$ $k = -4$ $T_n = 3^n - 4(n-1)(n-2)(n-3)$	✓ $a = 6$ ✓ method ✓ $b = -12$ ✓ $c = 9$ (4) ✓✓ $T_n = 3^n + k(n-1)(n-2)(n-3)$ ✓ substitution ✓ answer (4) [7]

November 2013

QUESTION 2

2.1 Given the geometric sequence: $7 ; x ; 63 ; ...$

Determine the possible values of x. (3)

2.2 The first term of a geometric sequence is 15. If the second term is 10, calculate:

2.2.1 T_{10} (3)

2.2.2 S_9 (2)

2.3 Given: $0 ; -\frac{1}{2} ; 0 ; \frac{1}{2} ; 0 ; \frac{3}{2} ; 0 ; \frac{5}{2} ; 0 ; \frac{7}{2} ; 0 ; ...$

Assume that this number pattern continues consistently.

2.3.1 Write down the value of the 191st term of this sequence. (1)

2.3.2 Determine the sum of the first 500 terms of this sequence. (4)

2.4 Given: $\sum_{k=2}^{20}(4x-1)^k$

2.4.1 Calculate the first term of the series $\sum_{k=2}^{20}(4x-1)^k$ if $x=1$. (2)

2.4.2 For which values of x will $\sum_{k=1}^{\infty}(4x-1)^k$ exist? (3)
[18]

QUESTION 3

3.1 Given the arithmetic sequence: $-3\,;1\,;5\,;\ldots;393$

3.1.1 Determine a formula for the n^{th} term of the sequence. (2)

3.1.2 Write down the 4^{th}, 5^{th}, 6^{th} and 7^{th} terms of the sequence. (2)

3.1.3 Write down the remainders when each of the first seven terms of the sequence is divided by 3. (2)

3.1.4 Calculate the sum of the terms in the arithmetic sequence that are divisible by 3. (5)

3.2 Consider the following pattern of dots:

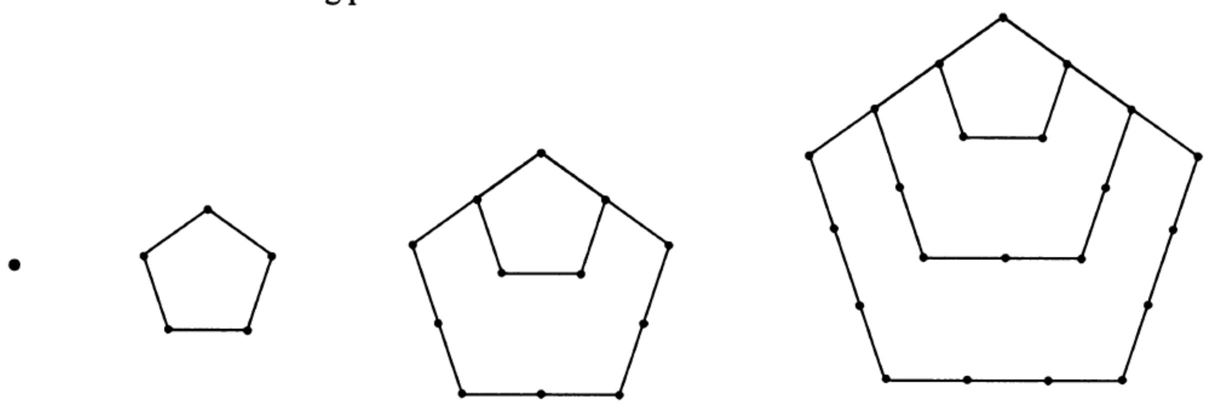


FIGURE 1 FIGURE 2 FIGURE 3 FIGURE 4

If T_n represents the total number of dots in FIGURE n, then $T_1=1$ and $T_2=5$. If the pattern continues in the same manner, determine:

3.2.1 T_5 (2)

3.2.2 T_{50} (5)
[18]

QUESTION/*VRAAG* 2

2.1	Given geometric sequence : $7\ ;x\ ;\ 63\ ...$ $\frac{T_2}{T_1}=\frac{T_3}{T_2}$ $\frac{x}{7}=\frac{63}{x}$ $x^2=441$ $x=\pm 21$	✓ $\frac{T_2}{T_1}=\frac{T_3}{T_2}$ / $\frac{x}{7}=\frac{63}{x}$ ✓ $x^2=441$ ✓ both answers (3)
	OR Given geometric sequence : $7\ ;x\ ;\ 63\ ...$ $\frac{T_2}{T_1}=\frac{T_3}{T_2}$ $\frac{x}{7}=\frac{63}{x}$ $x^2=441$ $x^2-441=0$ $(x-21)(x+21)=0$ $x=\pm 21$	✓ $\frac{T_2}{T_1}=\frac{T_3}{T_2}$ / $\frac{x}{7}=\frac{63}{x}$ ✓ $x^2=441$ ✓ both answers (3)
	OR $63=7r^2$ $r^2=9$ $r=\pm 3$ $x=\pm 21$	✓ $63=7r^2$ ✓ $r^2=9$ ✓ both answers (3)
2.2.1	$r=\frac{10}{15}=\frac{2}{3}$ $T_n=ar^{n-1}$ $T_{10}=15\left(\frac{2}{3}\right)^{10-1}$ $=\frac{2560}{6561}$ or $0{,}39$ **NOTE:** If the candidate rounds off early and gets $r=0{,}67$, then $T_{10}=0{,}41$: 3/3 marks	✓ $r=\frac{2}{3}$ ✓ correct subs into correct formula ✓answer (3)
	OR $r=\frac{10}{15}=\frac{2}{3}$ Expansion of the series $15+10+\frac{20}{3}+\frac{40}{9}+\frac{80}{27}+\frac{160}{81}+\frac{320}{243}+\frac{640}{729}+\frac{1280}{2187}+\frac{2560}{6561}$ $T_{10}=\frac{2560}{6561}$	✓ $r=\frac{2}{3}$ ✓ expansion of the series ✓answer (3)

2.2.2	$S_n = \frac{a(r^n - 1)}{r - 1}$ $S_9 = \frac{15\left(\left(\frac{2}{3}\right)^9 - 1\right)}{\frac{2}{3} - 1}$ $= \frac{95855}{2187}$ $= 43{,}83$ **OR** $S_n = \frac{a(1 - r^n)}{1 - r}$ $S_9 = \frac{15\left(1 - \left(\frac{2}{3}\right)^9\right)}{1 - \left(\frac{2}{3}\right)}$ $= \frac{95855}{2187}$ $= 43{,}83$	✓ correct substitution into correct formula ✓answer (2) ✓ substitution into correct formula ✓answer (2)
2.3.1	$T_{191} = 0$	✓answer (1)
2.3.2	Since the sum of all odd-positioned terms will be zero, need only consider the sum of the even-positioned terms, which form an arithmetic sequence, i.e. the sum of 250 **even** terms: *Omdat die som van al die terme in onewe posisies nul is, slegs nodig om die som van die terme in ewe posisies te oorweeg, wat 'n rekenkundige ry vorm, m.a.w. die som van 250* ***ewe*** *terme:* $S_{500} = \frac{250}{2}\left[2\left(-\frac{1}{2}\right) + (250 - 1)(1)\right]$ $= 31000$ **NOTE:** Breakdown: If $n = 500$ with $a = -\frac{1}{2}$ and $d = 1$ then $S_n = 124\ 500$: max 2/4 marks **OR** $S_{500} = \frac{125[2(-1) + 249(2)]}{2}$ $= 31000$ **OR**	✓$n = 250$ ✓$a = -\frac{1}{2}$ and $d = 1$ ✓ substitution into correct formula ✓ answer ✓$n = 125$ ✓$a = -1$ and $d = 2$ ✓ subs into correct formula ✓ answer (4)

	$\frac{3}{2}+\frac{5}{2}+\ldots$to 248 terms $=124\left[\frac{3}{2}+\frac{497}{2}\right]$ $=124\times 250$ $=31000$	✓$n=248$ ✓ subs into correct formula ✓ $\frac{3}{2}+\frac{247}{2}$ ✓ answer (4)
	OR	
	$\frac{3}{2}+\frac{5}{2}+\ldots$to 248 terms $=124[3+247]$ $=124\times 250$ $=31000$	✓$n=248$ ✓ subs into correct formula ✓ $3+247$ ✓ answer (4)
	OR	
	Sum = 0 + 4 + 8 +…to 125 terms $=\frac{125}{2}[0+(125-1)4]$ $=31000$	✓$n=125$ ✓$a=0$ and $d=4$ ✓ subs into correct formula ✓ answer (4)
2.4.1	$T_1=(4(1)-1)^2$ $=3^2$ $=9$ **NOTE:** If $k=1$, $T_1=3$: max 1/2 mark	✓ subs $x=1$ and $k=2$ ✓ answer (2)
2.4.2	$r=4x-1$ $-1<r<1$ $-1<4x-1<1$ $0<4x<2$ $0<x<\frac{1}{2}$ **NOTE:** Incorrect r : max 1/3 marks If candidate only writes down $4x-1$ and does nothing else: 0/3 marks	✓ $r=4x-1$ ✓ $-1<4x-1<1$ ✓ answer (3) **[18]**

QUESTION 2

2.1 A geometric sequence has $T_3 = 20$ and $T_4 = 40$.

Determine:

2.1.1 The common ratio (1)

2.1.2 A formula for T_n (3)

2.2 The following sequence has the property that the sequence of numerators is arithmetic and the sequence of denominators is geometric:

$$\frac{2}{1}\,;\,\frac{-1}{5}\,;\,\frac{-4}{25}\,;\ldots$$

2.2.1 Write down the FOURTH term of the sequence. (1)

2.2.2 Determine a formula for the n^{th} term. (3)

2.2.3 Determine the 500^{th} term of the sequence. (2)

2.2.4 Which will be the first term of the sequence to have a NUMERATOR which is less than –59? (3)

[13]

QUESTION 3

3.1 Given the arithmetic sequence: $w-3\ ;\ 2w-4\ ;\ 23-w$

3.1.1 Determine the value of w. (2)

3.1.2 Write down the common difference of this sequence. (1)

3.2 The arithmetic sequence 4 ; 10 ; 16 ; ... is the sequence of first differences of a quadratic sequence with a first term equal to 3.

Determine the 50$^{\text{th}}$ term of the quadratic sequence. (5)

[8]

QUESTION 4

In a geometric series, the sum of the first n terms is given by $S_n = p\left(1-\left(\frac{1}{2}\right)^n\right)$ and the sum to infinity of this series is 10.

4.1 Calculate the value of p. (4)

4.2 Calculate the second term of the series. (4)

[8]

QUESTION 2/*VRAAG* 2

2.1.1	$T_3 = 20$ and $T_4 = 40$ $r = \frac{T_4}{T_3} = 2$	✓ answer (1)
2.1.2	$T_n = ar^{n-1}$ $20 = a.2^{3-1}$ $a = 5$ $T_n = 5.2^{n-1}$ **OR** $40 = a.2^{4-1}$ $a = 5$ $T_n = 5.2^{n-1}$	✓subs into correct formula ✓ $a = 5$ ✓answer (3) ✓subs into correct formula ✓ $a = 5$ ✓answer (3)
2.2.1	$\frac{-7}{125}$	✓ answer (1)
2.2.2	$T_n = \frac{2+(n-1)(-3)}{(1).5^{n-1}}$ $T_n = \frac{5-3n}{5^{n-1}}$	✓ 5 ✓ 5^{n-1} ✓ $-3n$ (3)
2.2.3	$T_n = \frac{5-3n}{5^{n-1}}$ $T_{500} = \frac{5-3(500)}{5^{499}}$ $= \frac{-1495}{5^{499}}$	✓ numerator ✓ denominator (2)
2.2.4	$5-3n < -59$ $-3n < -64$ $n > 21{,}333...$ $n = 22$	✓ $5-3n < -59$ ✓ $n > 21{,}333...$ ✓ $n = 22$ (3) **[13]**

QUESTION/*VRAAG* 3

3.1.1	$w-3;\ 2w-4;\ 23-w$ $(2w-4)-(w-3)=(23-w)-(2w-4)$ $w-1=27-3w$ $4w=28$ $w=7$	✓ $(2w-4)-(w-3)$ $=(23-w)-(2w-4)$ ✓ $w=7$ (2)
3.1.2	Sequence is: 4 ; 10 ; 16 First difference / *Eerste verskil* = 6 **OR** $d=w-1$ $=6$	✓ answer (1) ✓ answer (1)
3.2	$T_{50}=3+(4+10+16+\ldots$ to 49 terms) $T_{50}=3+\frac{49}{2}[2(4)+(49-1)(6)]$ $=3+7252$ $=7255$ **OR** $2a=6$ $a=3$ $3a+b=4$ $3(3)+b=4$ $b=-5$ $a+b+c=3$ $3-5+c=3$ $c=5$ $T_n=3n^2-5n+5$ $T_{50}=3(50)^2-5(50)+5$ $=7255$	✓ $T_{50}=3+$ sum of 49 linear terms ✓ $a=4$ ✓ $n=49$ ✓ 7252(sum of 49 terms) ✓ answer (5) ✓ $a=3$ ✓ $b=-5$ ✓ $c=5$ ✓ substitution 50 ✓ answer (5) **[8]**

QUESTION/*VRAAG* 4

4.1	$S_n = p\left(1-\left(\frac{1}{2}\right)^n\right)$ $a = p\left[1-\left(\frac{1}{2}\right)^1\right]$ $= \frac{p}{2}$ $r = \frac{1}{2}$ $\therefore 10 = \frac{\frac{p}{2}}{1-\frac{1}{2}}$ $5 = \frac{p}{2}$ $p = 10$ **OR** $\left(\frac{1}{2}\right)^n \to 0 \text{ as } n \to \infty$ $\therefore S_\infty = p$ $p = 10$	✓ $a = \frac{p}{2}$ ✓ $r = \frac{1}{2}$ ✓ substitute in correct formula ✓ answer (4)
4.2	$r = \frac{1}{2}$ $\frac{a}{1-\frac{1}{2}} = 10$ $a = 5$ $T_2 = ar = \frac{5}{2}$ **OR**	✓ $r = \frac{1}{2}$ ✓ substitution ✓ $a = 5$ ✓ answer

	$S_n = 10 - 10.2^{-n}$ $a = T_1$ $= S_1$ $= 10 - 10.2^{-1}$ $= 10 - \frac{10}{2}$ $= 5$ $T_2 = S_2 - T_1$ $= 10 - 10.2^{-2} - 5$ $= 10 - \frac{10}{4} - 5$ $= \frac{5}{2}$	✓ $S_1 = 5$ ✓ $a = 5$ ✓ $T_2 = S_2 - T_1$ ✓ answer (4)
	OR $T_2 = S_2 - S_1$ $= p\left(1 - \left(\frac{1}{2}\right)^2\right) - p\left(1 - \frac{1}{2}\right)$ $= \frac{p}{4}$ $= \frac{10}{4}$ $= \frac{5}{2}$	✓ $T_2 = S_2 - S_1$ ✓ substitution ✓ $\frac{p}{4}$ ✓ answer (4) **[8]**

QUESTION 2

Given the arithmetic series: $2 + 9 + 16 + \ldots$ (to 251 terms).

2.1 Write down the fourth term of the series. (1)

2.2 Calculate the 251^{st} term of the series. (3)

2.3 Express the series in sigma notation. (2)

2.4 Calculate the sum of the series. (2)

2.5 How many terms in the series are divisible by 4? (4)

[12]

QUESTION 3

3.1 Given the quadratic sequence: $-1\ ; -7\ ; -11\ ; p\ ; \ldots$

3.1.1 Write down the value of p. (2)

3.1.2 Determine the n^{th} term of the sequence. (4)

3.1.3 The first difference between two consecutive terms of the sequence is 96. Calculate the values of these two terms. (4)

3.2 The first three terms of a geometric sequence are: $16\ ;\ 4\ ;\ 1$

3.2.1 Calculate the value of the 12^{th} term. (Leave your answer in simplified exponential form.) (3)

3.2.2 Calculate the sum of the first 10 terms of the sequence. (2)

3.3 Determine the value of: $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{5}\right)\ldots$ up to 98 factors. (4)

[19]

QUESTION/*VRAAG* 2

2.1	$T_4 = 23$	✓23 (1)
2.2	$T_{251} = a + (n-1)d$ $= 2 + (251-1)(7)$ $= 1752$	✓$a = 2$ and $d = 7$ ✓subst. into correct formula /*subt. in korrekte formule* ✓1752 (3)
2.3	$\sum_{n=1}^{251}(7n-5)$ **OR/*OF*** $\sum_{p=0}^{250}(7p+2)$	✓ general term/ *algemene term* ✓ complete answer /*volledige antwoord* (2) ✓ general term/ *algemene term* ✓ complete answer / *volledige antwoord* (2)
2.4	$S_n = \frac{n}{2}[a + l]$ $S_n = \frac{251}{2}[2 + 1752]$ $= 220127$ **OR/*OF*** $S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{251}{2}[2(2) + (251-1)(7)]$ $= 220127$	✓substitution/*substitusie* ✓220127 (2) ✓substitution/*substitusie* ✓220127 (2)
2.5	The new series/*Die nuwe reeks* is 16 + 44 + 72 + …+1 752 $16 + 28(n-1) = 1752$ $1736 = 28(n-1)$ $62 = n - 1$ $n = 63$ **OR/*OF*** 2 + 9 + <u>16</u> + 23 + 30 + 37 + <u>44</u> + 51 + ... + <u>1752</u> T_3 is divisible by /is deelbaar deur 4 Then $T_7, T_{11}, T_{15}, ..., T_{251}$ are divisible by 4, thus each 4th term is divisible by 4. Daarna is $T_7, T_{11}, T_{15}, ..., T_{251}$ deelbaar deur 4, d.w.s. elke 4de term is deelbaar deur 4. $\therefore$ number of terms divisible by 4 will be = $\frac{251-3}{4} + 1 = 63$ $\therefore$ aantal terme deelbaar deur 4 sal wees = $\frac{251-3}{4} + 1 = 63$ **OR/OF**	✓✓generating new series divisible by 4/ *vorming van nuwe reeks deelbaar deur 4* ✓ $T_n = 1752$ ✓ 63 (4) ✓T_3 is divisible by 4/ *is deelbaar deur 4* ✓identifying terms divisible by 4/ *identifiseer terme deelbaar deur 4* ✓ reasoning/*redenering* ✓63 (4)

	Position of terms divisible by 4: 3 ; 7 ; 11 ; …; 247; 251 $T_n = 4n - 1 = 251$ $4n = 252$ $n = 63$	✓✓ generating sequence involving position of terms/*vorming van reeks i.t.v. posisie van terme* ✓ $T_n = 251$ ✓ 63 (4) **[12]**

QUESTION/*VRAAG* 3

3.1.1	$-1\ ;\ -7\ ;\ -11\ ;\ p\ ;\ \ldots$ $-6 \qquad -4 \qquad p+11$ $2 \qquad 2$ $p+11-(-4)=2$ $p+15=2$ $p=-13$ **OR/*OF*** $-1\ ;\ -7\ ;\ -11\ ;\ p\ ;\ \ldots$ $-6 \qquad -4 \qquad p+11$ $2 \qquad 2$ $p+11=-2$ $p=-13$	✓$p+15=2$ ✓$p=-13$ (2) ✓first differences/ *eerste verskille* ✓$p=-13$ (2)
3.1.2	$2a=2$ $a=1$ $3a+b=-6$ $3(1)+b=-6$ $b=-9$ $a+b+c=-1$ $1-9+c=-1$ $c=7$ $T_n=n^2-9n+7$ **OR/*OF*** $T_n=T_1+(n-1)d_1+\frac{(n-1)(n-2)d_2}{2}$ $=-1+(n-1)(-6)+\frac{(n-1)(n-2)(2)}{2}$ $=-1-6n+6+\frac{2n^2-6n+4}{2}$ $=n^2-9n+7$	✓$a=1$ ✓$b=-9$ ✓$c=7$ ✓answer/*antwoord* (4) ✓formula/*formule* ✓substitution of first and second differences/*substitusie van eerste en tweede verskille* ✓simplification/*vereenvoudiging* ✓answer/*antwoord* (4)

	OR/*OF* $7;\ -1\ ;\ -7\ ;\ -11\ ;\ p\ ;\ \ldots$ $-8 \quad -6 \quad -4 \quad p+11$ $2 \quad 2 \quad 2$ $T_0 = 7 = c$ $2a = 2 \quad \therefore a = 1$ $3a + b = -6 \quad \therefore b = -9$ $T_n = n^2 - 9n + 7$	✓*c*-value/*c-waarde* ✓*a*-value/*a-waarde* ✓*b*-value/*b-waarde* ✓answer/*antwoord* (4)
	OR/OF $a = \frac{1}{2}(2) = 1$ $\therefore T_n = n^2 + bn + c$ $T_1 = -1 \ \therefore 1 + b + c = -1 \quad ....(1)$ $T_2 = -7 \ \therefore 4 + 2b + c = -7 \ ....(2)$ $(2) - (1): \quad 3 + b = -6$ $\therefore b = -9$ sub in $(1): c = 7$ $\therefore T_n = n^2 - 9n + 7$	✓*a*-value/*a-waarde* ✓*b*-value/*b-waarde* ✓*c*-value/*c-waarde* ✓answer/*antwoord* (4)

3.1.3	The sequence of first differences is/*Die reeks van eerste verskille is*: – 6 ; – 4 ; – 2 ; 0 ; ... $-6+(n-1)(2)=96$ $n=52$ $\therefore$ two terms are/*twee terme is*: $T_{52}=52^2-9(52)+7=2243$ $T_{53}=53^2-9(53)+7=2339$ **OR/OF** The sequence of first differences is/*Die reeks van eerste verskille is*: – 6 ; – 4 ; – 2 ; 0 ; ... The formula for the sequence of first differences/*Die formule vir die reeks van eerste verskille* is $T_n=2n-8$ 1st difference/*1ste verskil*: $2n-8=96$ $2n=104$ $n=52$ $\therefore$ two terms are/*twee terme is*: $T_{52}=52^2-9(52)+7=2243$ $T_{53}=53^2-9(53)+7=2339$ **OR/*OF*** $T_n-T_{n-1}=96$ $(n^2-9n+7)-\left[(n-1)^2-9(n-1)+7\right]=96$ $n^2-9n+7-n^2+2n-1+9n-9-7=96$ $2n=106$ $n=53$ $T_{52}=52^2-9(52)+7=2243$ $T_{53}=53^2-9(53)+7=2339$	✓$-6+(n-1)(2)=96$ ✓52 ✓2 243 ✓2 339 (4) ✓$2n-8=96$ ✓52 ✓2 243 ✓2 339 (4) ✓$T_n-T_{n-1}=96$ ✓53 ✓2 243 ✓2 339 (4)
	OR/OF $T_{n+1}-T_n=96$ $\left[(n+1)^2-9(n+1)+7\right]-\left[n^2-9n+7\right]=96$ $n^2+2n+1-9n-9+7-n^2+9n-7=96$ $2n=104$ $n=52$ $T_{52}=52^2-9(52)+7=2243$ $T_{53}=53^2-9(53)+7=2339$	✓$T_{n+1}-T_n=96$ ✓52 ✓2 243 ✓2 339 (4)

3.2.1	$T_{12} = 16\left(\frac{1}{4}\right)^{12-1}$ $= \frac{1}{4^9}$ or 4^{-9} or $\frac{1}{2^{18}}$ or 2^{-18}	✓ $a = 16$ and $r = \frac{1}{4}$ ✓subst. into correct formula/ *subt in korrekte formule* ✓ answer/*antwoord* (3)
3.2.2	$S_{10} = \frac{16\left(1-\left(\frac{1}{4}\right)^{10}\right)}{1-\frac{1}{4}}$ $= 21{,}33$ **OR/OF** $S_{10} = \frac{16\left(\left(\frac{1}{4}\right)^{10}-1\right)}{\frac{1}{4}-1}$ $= 21{,}33$	✓ substitution into correct formula /*substitusie in korrekte formule* ✓ answer/*antwoord* (2) ✓ substitution into correct formula /*substitusie in korrekte formule* ✓ answer/*antwoord* (2)
3.3	$\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\ldots\left(1+\frac{1}{99}\right)$ $= \left(\frac{\cancel{3}}{2}\right)\left(\frac{\cancel{4}}{\cancel{3}}\right)\left(\frac{\cancel{5}}{\cancel{4}}\right)\left(\frac{\cancel{6}}{\cancel{5}}\right)\ldots\left(\frac{100}{\cancel{99}}\right)$ $= \left(\frac{100}{2}\right)$ $= 50$ **OR/OF** $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\ldots\left(1+\frac{1}{99}\right)$ $T_1 = \left(1+\frac{1}{2}\right) = \frac{3}{2}$ $T_2 = \frac{3}{2}\left(1+\frac{1}{3}\right) = \frac{3}{2}\times\frac{4}{3} = 2$ $T_3 = 2\left(1+\frac{1}{4}\right) = 2\times\frac{5}{4} = \frac{5}{2}$ $\frac{3}{2}, 2, \frac{5}{2}$ ... is an arithmetic sequence with $a = \frac{3}{2}$ and $d = \frac{1}{2}$ $\therefore T_{98} = \frac{3}{2} + (98-1)\frac{1}{2}$ $= \frac{100}{2} = 50$	✓ improper fractions/ *onegte breuke* ✓ $\left(1+\frac{1}{99}\right)$ or $\left(\frac{100}{99}\right)$ ✓✓ answer/*antwoord* (4) ✓ $\left(1+\frac{1}{99}\right)$ ✓ giving the first three terms / *gee die eerste drie terme* ✓✓answer /*antwoord* (4) **[19]**

QUESTION 2

2.1 Given the arithmetic series: $18 + 24 + 30 + \ldots + 300$

2.1.1 Determine the number of terms in this series. (3)

2.1.2 Calculate the sum of this series. (2)

2.1.3 Calculate the sum of all the whole numbers up to and including 300 that are NOT divisible by 6. (4)

2.2 The first three terms of an infinite geometric sequence are 16, 8 and 4 respectively.

2.2.1 Determine the n^{th} term of the sequence. (2)

2.2.2 Determine all possible values of n for which the sum of the first n terms of this sequence is greater than 31. (3)

2.2.3 Calculate the sum to infinity of this sequence. (2)

[16]

QUESTION 3

3.1 A quadratic number pattern $T_n = an^2 + bn + c$ has a first term equal to 1.
The general term of the first differences is given by $4n + 6$.

3.1.1 Determine the value of a. (2)

3.1.2 Determine the formula for T_n. (4)

3.2 Given the series: $(1\times 2)+(5\times 6)+(9\times 10)+(13\times 14)+\ldots+(81\times 82)$

Write the series in sigma notation. (It is not necessary to calculate the value of the series.) (4)

[10]

QUESTION/VRAAG 2

2.1.1	$T_n = a + (n-1)d$ $300 = 18 + (n-1)6$ $300 = 18 + 6n - 6$ $6n = 288$ $n = 48$	✓ $a = 18$ and $d = 6$ ✓ $T_n = 300$ ✓ answer (3)
2.1.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{48}{2}[2(18) + 47(6)]$ $= 7632$	✓ substitution in formula ✓ answer (2)
2.1.3	Sum of all numbers from 1 to 300 / *Som van alle getalle van 1 tot 300* $= \frac{300}{2}[2(1) + 299(1)]$ $= \frac{300(301)}{2}$ $= 45150$ Sum of numbers not divisible by 6 / *Som van getalle wat nie deelbaar deur 6 is nie* $= 45150 - (7632 + 6 + 12)$ $= 37500$	✓ substitution ✓ answer ✓ $(7632 + 6 + 12)$ ✓ answer (4)
2.2.1	16, 8; 4; …… $r = \frac{1}{2}$ $T_n = ar^{n-1}$ $= 16\left(\frac{1}{2}\right)^{n-1}$ $= 2^4\left(2^{-n+1}\right)$ $= 2^{5-n}$	✓ $r = \frac{1}{2}$ ✓ answer (in any format) (2)
2.2.2	$16 + 8 + 4 + 2 + 1 + \frac{1}{2} = 31$ $S_5 = 31$ $n > 5$ or $n \geq 6$	✓ $16 + 8 + 4 + 2 + 1 + \frac{1}{2}$ ✓ $S_5 = 31$ ✓ $n > 5$ / $n \geq 6$ (3)

	OR $S_n = \frac{a(1-r^n)}{1-r}$ $31 < \frac{16\left(1-\frac{1}{2}^n\right)}{1-\frac{1}{2}}$ $31 < 32(1-2^{-n})$ $\frac{31}{32} - 1 < -2^{-n}$ $\frac{1}{32} > 2^{-n}$ $2^{-5} > 2^{-n}$ $n > 5$ or $n \geq 6$	✓ $S_n > 31$ ✓ simplification ✓ $n > 5$ / $n \geq 6$ (3)
2.2.3	$S_\infty = \frac{a}{1-r}$ $= \frac{16}{1-\frac{1}{2}}$ $= 32$ **OR** $16 + 8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} \ldots\ldots$ Answer gets <u>closer and closer to 32</u> the more terms gets added together *Antwoord beweeg nader en nader aan 32 hoe meer terme bymekaar getel word*	✓substitution of a and r ✓ answer (2) ✓expanding the series ✓ answer (2) **[16]**

QUESTION/VRAAG 3

3.1.1	1; x ; y; z….. $\quad$ $T_n = 4n + 6$ 10; 14; 18…… 1 $\quad$ x $\quad$ y 10 $\quad$ 14 4 $2a = 4$ $a = 2$ **OR** $T_n = 4n + 6$ $d = 4$ $2a = 4$ $a = 2$	2^{nd} difference = 4 ✓ $2a = 4$ ✓ $a = 2$ (2) ✓ $2a = 4$ ✓ $a = 2$ (2)
3.1.2	1 $\quad$ x $\quad$ y 10 $\quad$ 14 4 $3a + b = 10$ $6 + b = 10$ $b = 4$ $a + b + c = 1$ $2 + 4 + c = 1$ $c = -5$ $T_n = 2n^2 + 4n - 5$	✓ 1^{st} differences 10; 14; 18…… ✓ $3a + b = 10$ ✓ $a + b + c = 1$ ✓ $T_n = 2n^2 + 4n - 5$ (4)

3.2	Consider the sequence made up by the first factors of each term: *Beskou die ry wat deur die eerste faktore van elke term gevorm word:* 1; 5; 9; 13; … 81 An arithmetic sequence / *rekenkundige ry*: $T_n = a + (n-1)d$ $= 1 + (n-1)4$ $= 4n - 3$	✓ $T_n = 4n - 3$
	To find the no. of terms: $81 = 4n - 3$ *Aantal terme:* $4n = 84$ $\therefore n = 21$	✓no. of terms
	The second factor is 1 more than the first factor / *Tweede faktor is 1 meer as die eerste faktor:* $T_n = 4n - 3 + 1$ $= 4n - 2$	✓ $T_n = 4n - 2$
	OR Consider the sequence made up by the second factors of each term: *Beskou die ry wat deur die tweede faktore van elke term gevorm word:* 2; 6; 10; 14; …82 Also an arithmetic sequence / *rekenkundige ry*: $T_n = a + (n-1)d$ $= 2 + (n-1)4$ $= 4n - 2$ **Answer only full marks**	✓ $T_n = 4n - 2$
	In sigma notation: $\sum_{n=1}^{21}(4n-3)(4n-2)$ **or** $\sum_{n=1}^{21}2(4n-3)(2n-1)$ **or** $\sum_{n=1}^{21}(16n^2 - 20n + 6)$	✓answer in sigma notation (4) **[10]**

QUESTION 2

2.1 Prove that in any arithmetic series in which the first term is a and whose constant difference is d, the sum of the first n terms is $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$. (4)

2.2 Calculate the value of $\sum_{k=1}^{50}(100-3k)$. (4)

2.3 A quadratic sequence is defined with the following properties:

$$T_2 - T_1 = 7$$
$$T_3 - T_2 = 13$$
$$T_4 - T_3 = 19$$

2.3.1 Write down the value of:

(a) $T_5 - T_4$ (1)

(b) $T_{70} - T_{69}$ (3)

2.3.2 Calculate the value of T_{69} if $T_{89} = 23\,594$. (5)

[17]

QUESTION 3

Consider the infinite geometric series: $45 + 40{,}5 + 36{,}45 + \ldots$

3.1 Calculate the value of the TWELFTH term of the series (correct to TWO decimal places). (3)

3.2 Explain why this series converges. (1)

3.3 Calculate the sum to infinity of the series. (2)

3.4 What is the smallest value of n for which $S_\infty - S_n < 1$? (5)

[11]

QUESTION/*VRAAG* 2

2.1	$S_n = a + (a+d) + (a+2d) + \ldots + a + (n-1)d$ $S_n = a + (n-1)d + a + (n-2)d + a + (n-3)d + \ldots + a$ $2S_n = n(2a + (n-1)d)$ $S_n = \frac{n}{2}[2a + (n-1)d]$	✓ first series/*eerste reeks* ✓series reversed/*reeks omgekeer* ✓ sum/*som* ✓ division/*deling* (4)
2.2	$\sum_{k=1}^{50}(100-3k) = 97 + 94 + 91 + \ldots$ $T_1 = a = 97$ $d = -3$ $n = 50 - 1 + 1 = 50$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{50}{2}[2(97) + 49(-3)]$ $= 1175$	✓$a = 97$ ✓$d = -3$ ✓$n = 50$ ✓ answer/*antwoord* (4)
	OR/*OF* $T_1 = a = 97$ $l = 100 - 3(50) = -50$ $n = 50 - 1 + 1 = 50$ $S_n = \frac{n}{2}[a + l]$ $= \frac{50}{2}[97 - 50]$ $= 1175$	✓$a = 97$ ✓$l = -50$ ✓$n = 50$ ✓ answer/*antwoord* (4)

2.3.1 (a)	$T_5 - T_4 = 25$	✓ answer/*antwoord* (1)
2.3.1 (b)	$T_{70} - T_{69} = 7 + (69-1)(6)$ $= 415$	✓ $n = 69$ ✓ $7+(69-1)(6)$ ✓ answer/*antw.* (3)
2.3.2	$T_{89} - T_{69} = (T_{70} - T_{69}) + (T_{71} - T_{70}) + \ldots + (T_{89} - T_{88})$ $= 415 + 421 + \ldots$to 20 terms $= \frac{20}{2}[2(415)+19(6)]$ $= 9440$ $T_{69} = T_{89} -$(sum of the differences from/*som van die verskille van* T_{69} to T_{89}) $T_{69} = 23594 - 9440$ $= 14154$	✓ expansion/*uitbreiding* ✓ $n = 20$ ✓ method/*metode* ✓ $a = 415$ ✓ answer/*antwoord* (5)
	OR/*OF* 7 13 19 25 6 6 6 $\therefore 2a = 6$ $a = 3$ $3a + b = 7$ $b = -2$ $T_{89} = 3(89)^2 - 2(89) + c = 23594$ $\therefore c = 9$ $\therefore T_n = 3n^2 - 2n + 9$ $\therefore T_{69} = 3(69)^2 - 2(69) + 9$ $\therefore T_{69} = 14154$	 ✓ a and/*en* b ✓ T_{89} (subst $n = 89$) ✓ T_n ✓substitution/*substitusie* ✓ answer/*antwoord* (5)

	OR/*OF* 7 13 19 25 6 6 6 $\therefore 2a = 6$ $a = 3$ $7 - 6 = 1$ $T_1 - T_0 = 1$ $a + b + c - c = 1$ $3 + b = 1$ $b = -2$ $T_{89} = 3(89)^2 - 2(89) + c = 23594$ $\therefore c = 9$ $\therefore T_n = 3n^2 - 2n + 9$ $\therefore T_{69} = 3(69)^2 - 2(69) + 9$ $\therefore T_{69} = 14154$	✓ a and/*en* b ✓ T_{89} (subst $n = 89$) ✓ T_n ✓substitution/*substitusie* ✓ answer/*antwoord* (5) **[17]**
	OR/*OF* $T_{n+1} - T_n = 7 + 6(n-1)$ $\therefore T_{89} - T_1 = \sum_{n=1}^{88}(T_{n+1} - T_n)$ $= \frac{n}{2}[2a + (n-1)d]$ $= \frac{88}{2}[14 + 87 \times 6]$ $= 23584$ $\therefore T_1 = 23594 - 23584 = 10$ $\therefore T_{69} - 10 = \sum_{n=1}^{68}(T_{n+1} - T_n)$ $= 34(15 + 67 \times 6) = 14144$ $\therefore T_{69} = 14154$	✓formula/*formule* ✓value of/*waarde van* S_{88} ✓first term value/ *eerste term waarde* ✓substitution/*substitusie* ✓answer/*antwoord* (5) **[17]**

QUESTION 3

3.1	$r=\frac{40{,}5}{45}=0{,}9$ $T_{12}=45(0{,}9)^{12-1}$ $=14{,}12147682...$ $=14{,}12$	✓$r=0{,}9$ ✓ substitution into correct formula/*substitusie in korrekte formule* ✓ answer/*antwoord* (3)
3.2	$r=0{,}9$ $-1<0{,}9<1$	✓answer/*antwoord* (1)
3.3	$S_\infty=\frac{45}{1-0{,}9}$ $S_\infty=450$	✓ substitution/*substitusie* ✓ 450 (2)
3.4	$S_\infty-S_n<1$ $S_\infty-S_n=450-\frac{45\left(1-(0{,}9)^n\right)}{1-0{,}9}$ $S_\infty-S_n=450-450\left(1-(0{,}9)^n\right)$ $450(0{,}9)^n<1$ $(0{,}9)^n<\frac{1}{450}$ $\log(0{,}9)^n<\log\frac{1}{450}$ $n.\log(0{,}9)<\log\frac{1}{450}$ $n>\frac{\log\frac{1}{450}}{\log(0{,}9)}$ $n>57{,}98...$ Smallest value/*Kleinste waarde:* $n = 58$	✓ $450-\frac{45\left(1-(0{,}9)^n\right)}{1-0{,}9}$ ✓ $(0{,}9)^n=\frac{1}{450}$ ✓ introducing/*gebruik* logs ✓making n the subject/*maak n die onderwerp* ✓$n=58$ (5) **[11]**

QUESTION 2

The following geometric sequence is given: 10 ; 5 ; 2,5 ; 1,25 ; ...

2.1 Calculate the value of the 5^{th} term, T_5, of this sequence. (2)

2.2 Determine the n^{th} term, T_n, in terms of n. (2)

2.3 Explain why the infinite series $10 + 5 + 2,5 + 1,25 + ...$ converges. (2)

2.4 Determine $S_\infty - S_n$ in the form ab^n, where S_n is the sum of the first n terms of the sequence. (4)

[10]

QUESTION 3

Consider the series: $S_n = -3 + 5 + 13 + 21 + ...$ to n terms.

3.1 Determine the general term of the series in the form $T_k = bk + c$. (2)

3.2 Write S_n in sigma notation. (2)

3.3 Show that $S_n = 4n^2 - 7n$. (3)

3.4 Another sequence is defined as:

$$Q_1 = -6$$
$$Q_2 = -6 - 3$$
$$Q_3 = -6 - 3 + 5$$
$$Q_4 = -6 - 3 + 5 + 13$$
$$Q_5 = -6 - 3 + 5 + 13 + 21$$

3.4.1 Write down a numerical expression for Q_6. (2)

3.4.2 Calculate the <u>value</u> of Q_{129}. (3)

[12]

QUESTION/*VRAAG* 2

2.1	$r=\dfrac{T_2}{T_1}$ $=\dfrac{5}{10}$ $=\dfrac{1}{2}$ $T_5=1{,}25\left(\dfrac{1}{2}\right)$ $=\dfrac{5}{8}$ or $0{,}625$ **OR/*OF*** $T_5=10\left(\dfrac{1}{2}\right)^4$ $=\dfrac{5}{8}$ or $0{,}625$	✓ $r=\dfrac{1}{2}$ ✓answer (2)
2.2	$T_n=10\left(\dfrac{1}{2}\right)^{n-1}$	✓substitutes $a=10$ into GP formula ✓substitutes $r=\dfrac{1}{2}$ into GP formula (2)
2.3	For convergence/*Om te konvergeer* $-1<r<1$ Since/*Aangesien* $r=\dfrac{1}{2}$ and/*en* $-1<\dfrac{1}{2}<1$ the sequence converges/*die ry konvergeer*	✓ $-1<r<1$ ✓ show that $r=\dfrac{1}{2}$ is $-1<r<1$ (2)
2.4	$S_\infty - S_n=\dfrac{a}{1-r}-\dfrac{a(1-r^n)}{1-r}$ $=\dfrac{10}{1-\frac{1}{2}}-\dfrac{10\left(1-\frac{1}{2}^n\right)}{1-\frac{1}{2}}$ $=20-20\left(1-\frac{1}{2}^n\right)$ $=20-20+20\left(\dfrac{1}{2}\right)^n$ $=20\left(\dfrac{1}{2}\right)^n$ **OR/*OF***	✓ $\dfrac{10}{1-\frac{1}{2}}$ ✓ $\dfrac{10\left(1-\frac{1}{2}^n\right)}{1-\frac{1}{2}}$ ✓ $20\left(1-\frac{1}{2}^n\right)$ ✓answer (4) ✓constructing the series

	$S_\infty - S_n = T_{n+1} + T_{n+2} + T_{n+3} + \ldots$	✓
	$= 10\left(\frac{1}{2}\right)^n\left[1 + \frac{1}{2} + \frac{1}{4} + \ldots\right]$	$10\left(\frac{1}{2}\right)^n\left[1 + \frac{1}{2} + \frac{1}{4} + \ldots\right]$
	$= 10\left(\frac{1}{2}\right)^n\left[\frac{1}{1-\frac{1}{2}}\right]$	✓ $\frac{1}{1-\frac{1}{2}}$
	$= 20\left(\frac{1}{2}\right)^n$	✓answer
		(4)
	OR/*OF*	
	$S_\infty - S_n = \frac{a}{1-r} - \frac{a(1-r^n)}{1-r}$	✓ $\frac{a-a+ar^n}{1-r}$
	$= \frac{a-a+ar^n}{1-r}$	✓ $\frac{ar^n}{1-r}$
	$= \frac{ar^n}{1-r}$	✓ $\frac{10\left(\frac{1}{2}\right)^n}{\frac{1}{2}}$
	$= \frac{10\left(\frac{1}{2}\right)^n}{\frac{1}{2}}$	✓answer
	$= 20\left(\frac{1}{2}\right)^n$	(4) **[10]**

QUESTION/*VRAAG* 3

3.1	$d = 8$ $T_k = a + (k-1)d$ $= -3 + (k-1)(8)$ $= -3 + 8k - 8$ $= 8k - 11$	✓ d value ✓answer (2)
3.2	$\sum_{k=1}^{n}(8k-11)$ **OR/*OF*** $\sum_{k=0}^{n-1}(8(k+1)-11) = \sum_{k=0}^{n-1}(8k-3)$	✓for general term ✓lower and upper values in sigma notation (2)
3.3	$S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{n}{2}[2(-3) + (n-1)(8)]$ $= \frac{n}{2}[-6 + 8n - 8]$ $= \frac{n}{2}[8n - 14]$ $= n(4n-7)$ $= 4n^2 - 7n$ **OR/*OF*** $S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{n}{2}[2(-3) + (n-1)(8)]$ $= \frac{n}{2}[-6 + 8n - 8]$ $= \frac{n}{2}[8n - 14]$ $= 4n^2 - 7n$ **OR/*OF*** $S_n = \frac{n}{2}[a + l]$ $= \frac{n}{2}[-3 + 8n - 11]$ $= \frac{n}{2}[8n - 14]$ $= 4n^2 - 7n$	✓formula ✓substitution ✓ $\frac{n}{2}[8n-14]$ (3) ✓formula ✓substitution ✓ $\frac{n}{2}[8n-14]$ (3) ✓formula ✓substitution ✓ $\frac{n}{2}[8n-14]$ (3)

	OR/*OF* S_1 S_2 S_3 S_4 -3 2 15 36 5 13 21 8 8 $S_n = an^2 + bn + c$ $a = \frac{8}{2}$ $a = 4$ $S_1 = 4 + b + c = -3$ $b + c = -7$ ....(1) $S_2 = 16 + 2b + c = 2$ $2b + c = -14$..............(2) $b = -7$................(2)$-$(1) $c = 0$ Hence $S_n = 4n^2 - 7n$	$S_2 = -3 + 5 = 2$ $S_3 = 2 + 13 = 15$ $S_4 = 15 + 21$ ✓calculates S_1, S_2, S_3 and S_4, ✓ $a = 4$ ✓solves simultaneously for b and c. (3)
3.4.1	$Q_6 = -6 - 3 + 5 + 13 + 21 + 29$	✓✓ answer (2)
3.4.2	$Q_{129} = -6 + S_{128}$ $= -6 + 4(128)^2 - 7(128)$ $= 64634$ **OR/*OF*** Q_1 Q_2 Q_3 Q_4 -6 -9 -4 9 -3 5 13 8 8 8 $Q_n = an^2 + bn + c$ $a = 4$ $Q_1 = 4 + b + c = -6$ $b + c = -10$ ....(1) $Q_2 = 16 + 2b + c = -9$ $2b + c = -25$...............(2) $b = -15$................(2)$-$(1) $c = 5$ Hence $Q_n = 4n^2 - 15n + 5$ $Q_{129} = 4(129)^2 - 15(129) + 5$ $= 64\ 634$	✓✓ $-6 + 4(128)^2 - 7(128)$ ✓answer (3) ✓$a = 4$ ✓ $Q_n = 4n^2 - 15n + 5$ ✓answer (3) **[12]**

QUESTION 2

2.1 Given the following quadratic sequence: $-2\ ;\ 0\ ;\ 3\ ;\ 7\ ;\ ...$

2.1.1 Write down the value of the next term of this sequence. (1)

2.1.2 Determine an expression for the n^{th} term of this sequence. (5)

2.1.3 Which term of the sequence will be equal to 322? (4)

2.2 Consider an arithmetic sequence which has the second term equal to 8 and the fifth term equal to 10.

2.2.1 Determine the common difference of this sequence. (3)

2.2.2 Write down the sum of the first 50 terms of this sequence, using sigma notation. (2)

2.2.3 Determine the sum of the first 50 terms of this sequence. (3)

[18]

QUESTION 3

Chris bought a bonsai (miniature tree) at a nursery. When he bought the tree, its height was 130 mm. Thereafter the height of the tree increased, as shown below.

INCREASE IN HEIGHT OF THE TREE PER YEAR		
During the first year	**During the second year**	**During the third year**
100 mm	70 mm	49 mm

3.1 Chris noted that the sequence of height increases, namely 100 ; 70 ; 49 ..., was geometric. During which year will the height of the tree increase by approximately 11,76 mm? (4)

3.2 Chris plots a graph to represent the height $h(n)$ of the tree (in mm) n years after he bought it. Determine a formula for $h(n)$. (3)

3.3 What height will the tree eventually reach? (3)

[10]

QUESTION/*VRAAG* 2

2.1.1	$-2 \quad 0 \quad 3 \quad 7 \quad \mathbf{12}$ $2 \quad 3 \quad 4 \quad \mathbf{5}$ $1 \quad 1 \quad 1$ The next term of the sequence is 12./*Die volgende term in die ry is 12.*	✓answer (1)
2.1.2	$2a = 1$ $a = \frac{1}{2}$ $3a + b = T_2 - T_1$ $3\left(\frac{1}{2}\right) + b = 2$ $b = \frac{1}{2}$ $a + b + c = T_1$ $\frac{1}{2} + \frac{1}{2} + c = -2$ $c = -3$ $\therefore T_n = \frac{1}{2}n^2 + \frac{1}{2}n - 3$ **OR/*OF***	✓value of a ✓ $3\left(\frac{1}{2}\right) + b = 2$ ✓value of b ✓ $\frac{1}{2} + \frac{1}{2} + c = -2$ ✓ value of c (5)

	$2a=1$ $a=\frac{1}{2}$ $T_n=an^2+bn+c$ $-2=\frac{1}{2}+b+c$..............T_1 $b+c=-\frac{5}{2}$........................line 1 $0=2+2b+c$...............T_2 $2b+c=-2$........................line 2 line 2 – line 1: $b=\frac{1}{2}$ substitute in line 1 or substitute in line 2 $\frac{1}{2}+c=-\frac{5}{2}$ $2\left(\frac{1}{2}\right)+c=-2$ $c=-3$ $\therefore T_n=\frac{1}{2}n^2+\frac{1}{2}n-3$	✓value of a ✓ $-2=\frac{1}{2}+b+c$ ✓ $0=2+2b+c$ ✓value of b ✓ value of c	(5)
	OR/*OF* $T_n=T_1+(n-1)d_1+\frac{(n-1)(n-2)}{2}d_2$ $=-2+(n-1)(2)+\frac{(n-1)(n-2)}{2}(1)$ $=-2+2n-2+(n^2-3n+2)(\frac{1}{2})$ $=-2+2n-2+\frac{1}{2}n^2-\frac{3}{2}n+1$ $=\frac{1}{2}n^2+\frac{1}{2}n-3$	✓formula ✓substitution ✓ value of a ✓value of b ✓ value of c	(5)
	OR/*OF* $2a=1$ $a=\frac{1}{2}$ $3a+b=T_2-T_1$ $3\left(\frac{1}{2}\right)+b=2$ $b=\frac{1}{2}$ $T_0=c=-3$ $\therefore T_n=\frac{1}{2}n^2+\frac{1}{2}n-3$ **OR/*OF***	✓value of a ✓ $3\left(\frac{1}{2}\right)+b=2$ ✓value of b ✓ $T_0=c$ ✓ value of c	(5)

	Since $T_2 = 0, (n-2)$ is a factor of T_n $T_n = an^2 + bn + c$ $= a(n-2)(n-k)$ $T_1 = -2 = a(1-2)(1-k)$ $-2 = -a(1-k)$ $a = \frac{2}{1-k}$ $T_3 = 3 = a(3-2)(3-k)$ $3 = a(3-k)$ $a = \frac{3}{3-k}$ $\frac{2}{1-k} = \frac{3}{3-k}$ $2(3-k) = 3(1-k)$ $6-2k = 3-3k$ $k = -3$ $a = \frac{1}{2}$ $T_n = \frac{1}{2}(n-2)(n+3)$ $= \frac{1}{2}n^2 + \frac{1}{2}n - 3$	✓ $T_n = a(n-2)(n-k)$ ✓ $-2 = a(1-2)(1-k)$ ✓ $3 = a(3-2)(3-k)$ ✓value of k ✓value of a (5)
2.1.3	$\frac{1}{2}n^2 + \frac{1}{2}n - 3 = 322$ $n^2 + n - 6 = 644$ $n^2 + n - 650 = 0$ $n = \frac{-1 \pm \sqrt{1^2 - 4(1)(650)}}{2}$ $n = 25$ or $n = -26$ The 25th term has a value of 322./*Die 25ste term se waarde is 322.* **OR/*OF*** $\frac{1}{2}n^2 + \frac{1}{2}n - 3 = 322$ $n^2 + n - 6 = 644$ $n^2 + n - 650 = 0$ $(n-25)(n+26) = 0$ $n = 25$ or $n = -26$ The 25th term has a value of 322./*Die 25ste term se waarde is 322.* **OR/*OF***	✓ $\frac{1}{2}n^2 + \frac{1}{2}n - 3 = 322$ ✓standard form ✓substitution into quadratic formula ✓answer (4) ✓ $\frac{1}{2}n^2 + \frac{1}{2}n - 3 = 322$ ✓standard form ✓factors ✓answer (4)

	$\frac{1}{2}n^2+\frac{1}{2}n-3=322$ $n^2+n-6=644$ $(n+3)(n-2)=23\times 28$ $n-2=23$ $n=25$	✓ $\frac{1}{2}n^2+\frac{1}{2}n-3=322$ ✓ $(n+3)(n-2)$ ✓ 23×28 ✓answer (4)
2.2.1	$T_2: \quad a+d=8$ $T_5: \quad a+4d=10$ $T_5-T_2: \quad 3d=2$ $d=\frac{2}{3}$	✓ $a+d=8$ ✓ $a+4d=10$ ✓answer (3)
2.2.2	$T_1=T_2-d$ $=8-\frac{2}{3}$ $=\frac{22}{3}$ $T_n=a+(n-1)d$ $=\frac{22}{3}+(n-1)\frac{2}{3}$ $=\frac{2n+20}{3}$ $S_{50}=\sum_{n=1}^{50}\left(\frac{22}{3}+(n-1)\frac{2}{3}\right)$ **OR/*OF*** $S_{50}=\sum_{n=1}^{50}\left(\frac{2n+20}{3}\right)$	✓ $T_1=\frac{22}{3}$ ✓answer (2) (2)
2.2.3	$S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{50}=\frac{50}{2}\left[2\left(\frac{22}{3}\right)+(50-1)\left(\frac{2}{3}\right)\right]$ $=\frac{3550}{3}$	✓correct substitution into correct formula ✓✓answer (3) **[18]**

QUESTION/*VRAAG* 3

3.1	$r=\frac{70}{100}$ $=\frac{7}{10}$ $T_n=ar^{n-1}$ $11{,}76=100\left(\frac{7}{10}\right)^{n-1}$ $\left(\frac{7}{10}\right)^{n-1}=\frac{11{,}76}{100}$ $n-1=\log_{\frac{7}{10}}\left(\frac{11{,}76}{100}\right)$ $n-1=6$ $n=7$ During the 7th year/*In die 7de jaar* **OR/*OF*** $r=\frac{70}{100}$ $=\frac{7}{10}$ $T_n=ar^{n-1}$ $11{,}76=100(0{,}7)^{n-1}$ $0{,}7^{n-1}=\frac{11{,}76}{100}$ $=0{,}1176$ $(n-1)\log 0{,}7=\log 0{,}1176$ $n-1=\frac{\log 0{,}1176}{\log 0{,}7}$ $n-1=6$ $n=7$ During the 7th year/*In die 7de jaar*	✓value of r ✓substitution in formula for T_n ✓use of logarithms ✓answer (4) ✓value of r ✓substitution in formula for T_n ✓use of logarithms ✓answer (4)
3.2	$h(n)=130+(100+70+49+\ldots\text{to } n \text{ terms})$ $=130+\frac{100\left(1-(0{,}7)^n\right)}{1-0{,}7}$ $=130+\frac{100\left(1-(0{,}7)^n\right)}{0{,}3}$	✓130 ✓ $100+70+49+\ldots\text{to } n \text{ terms}$ ✓answer (3)

3.3	Eventual height of the tree/*Uiteindelike hoogte van die boom* $= 130 + \frac{100}{1-0,7}$ $= 463,33$ mm OR $\frac{1390}{3}$ mm	✓✓$130 + \frac{100}{1-0,7}$ ✓answer (3) **[10]**

QUESTION 2

Given the finite arithmetic sequence: $5 \ ; \ 1 \ ; \ -3 \ ; \ \ldots \ ; \ -83 \ ; \ -87$

2.1 Write down the fourth term (T_4) of the sequence. (1)

2.2 Calculate the number of terms in the sequence. (3)

2.3 Calculate the sum of all the negative numbers in the sequence. (3)

2.4 Consider the sequence: $5 \ ; \ 1 \ ; \ -3 \ ; \ \ldots \ ; \ -83 \ ; \ -87 \ ; \ \ldots \ ; \ -4\,187$
Determine the number of terms in this sequence that will be exactly divisible by 5. (4)

[11]

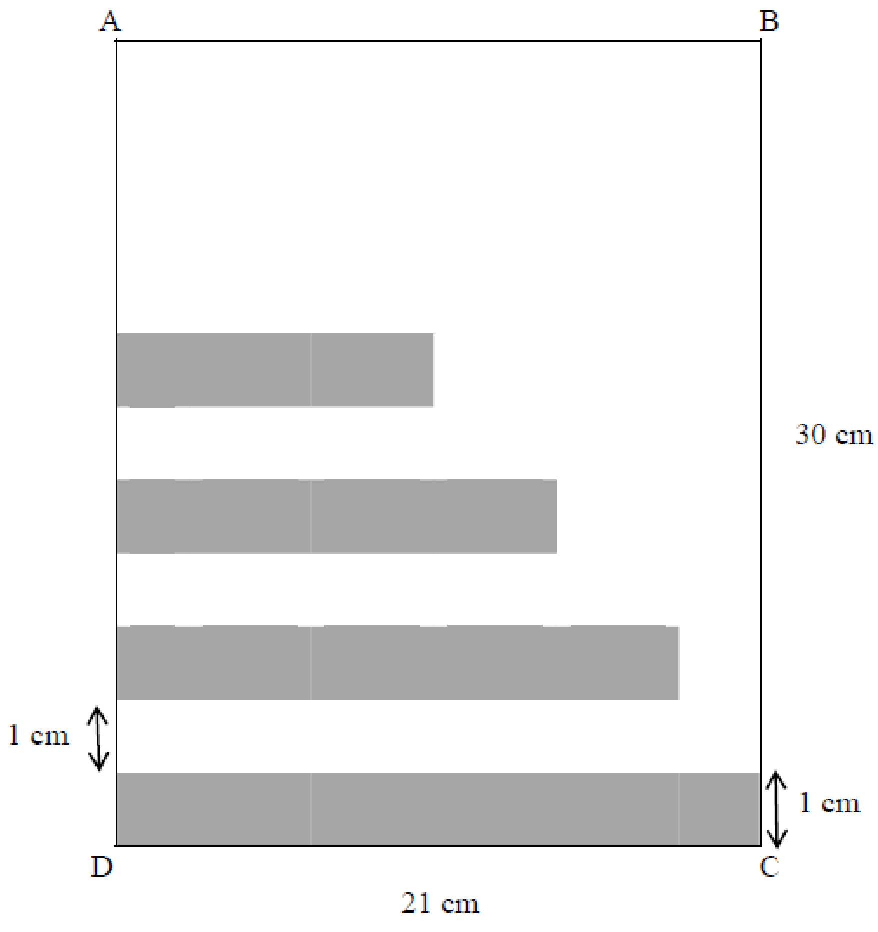
QUESTION 3

3.1 The first four terms of a quadratic number pattern are $-1\ ;\ x\ ;\ 3\ ;\ x+8$

3.1.1 Calculate the value(s) of x. (4)

3.1.2 If $x = 0$, determine the position of the first term in the quadratic number pattern for which the sum of the first n first differences will be greater than 250. (4)

3.2 Rectangles of width 1 cm are drawn from the edge of a sheet of paper that is 30 cm long such that there is a 1 cm gap between one rectangle and the next. The length of the first rectangle is 21 cm and the length of each successive rectangle is 85% of the length of the previous rectangle until there are rectangles drawn along the entire length of AD. Each rectangle is coloured grey.



3.2.1 Calculate the length of the 10^{th} rectangle. (3)

3.2.2 Calculate the percentage of the paper that is coloured grey. (4)

[15]

QUESTION/*VRAAG* 2

2.1	$T_4 = -7$	✓ -7 (1)
2.2	$T_n = a + (n-1)d$ $-87 = 5 + (n-1)(-4)$ $-87 = 5 - 4n + 4$ $4n = 96$ $n = 24$ **OR/OF** $-4n + 9 = -87$ $-4n = -96$ $n = 24$	✓ $a = 5$ and $d = -4$ ✓ $-87 = 5 + (n-1)(-4)$ ✓ $n = 24$ (3) ✓ $-4n + 9$ ✓ $-4n + 9 = -87$ ✓ $n = 24$ (3)
2.3	$-3; -7; ......; -87$ $S_n = \frac{n}{2}[a + T_n]$ $S_{22} = \frac{22}{2}[-3 - 87]$ $= -990$ **OR/OF**	✓ $n = 22$ ✓ $a = -3$ ✓ answer (3)

	$-3;-7;......;-87$ $S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{22}=\frac{22}{2}[2(-3)+(22-1)(-4)]$ $=-990$ **OR/OF** All negative terms can be written down and added to get the answer of –990./*Alle negatiewe terme kan neergeskryf word en dan bymekaar getel word om –990 te kry.* **OR/OF** Sum $=S_{24}-(5+1)$ $=\frac{24}{2}[5-87]-6$ $=-990$	✓ $n=22$ ✓ $a=-3$ ✓ answer (3) ✓ $a=-3$ ✓✓ answer (3) ✓ $\frac{24}{2}[5-87]$ ✓ -6 ✓ answer (3)
2.4	$5;-15;-35......$ $d=-20$ $T_n=-20n+25$ Last term in the sequence divisible by 5 is:/*Laaste term in die ry deelbaar deur 5 is:* $-4187+4(3)$ $=-4175$ $T_n=-20n+25$ $-4175=-20n+25$ $20n=4200$ $n=210$ There will be 210 terms in the sequence that is divisible by 5./*Daar is 210 terme in die ry deelbaar deur 5.* **OR/OF**	✓ $d=-20$ ✓ $T_n=-20n+25$ ✓ $-4175=-20n+25$ ✓ $n=210$ (4)

	$5\ ;\ 1\ ;\ -3\ ;\ \ldots\ ;\ -83\ ;\ -87;\ \ldots\ldots;\ -4187$ $T_n = -4n + 9$ $-4187 = -4n + 9$ $4n = 4196$ $n = 1049$ There are 1049 terms in the sequence./*Daar is* 1049 *terme in die ry.*	✓ $-4n + 9 = -4187$ ✓ $n = 1049$
	$T_1\ ;\ T_6\ ;\ T_{11}\ ;\ T_{16}\ \ldots$ are divisible by 5./*is deelbaar deur* 5. The largest integer value of k such that $5k - 4 \leq 1049$ $5k \leq 1053$ $k \leq 210{,}6$ $k = 210$	✓ $5k - 4 \leq 1049$ ✓ $k = 210$ (4)
	OR/***OF*** **5** ; 1 –3 ; –7 ; … ; **–4175**; –4179 ;–4183 ; –4187 $T_n = a + (n-1)d$ $-4175 = 5 + (n-1)(-4)$ $-4180 = -4(n-1)$ $n = 1046$	✓ $d = -4$ ✓ $-4175 = -4n + 9$ ✓ 1046
	Number of terms divisible by 5 $= \dfrac{1046-1}{5} + 1$ $= 210$	✓ $n = 210$ (4) **[11]**

QUESTION/*VRAAG* 3

3.1.1	$-1\ ;\ x\ ;\ 3\ ;\ x+8\ ;\ \ldots$ $-1 \quad x \quad 3 \quad x+8$ $x+1 \quad 3-x \quad x+5$ $-2x+2 \quad 2x+2$ $-2x+2=2x+2$ $4x=0$ $x=0$	✓ $x+1;3-x$ and $x+5$ ✓ calculating second differences ✓ $-2x+2=2x+2$ ✓ $x=0$ (4)
3.1.2	First differences/*Eerste verskille*: $1\ ;\ 3\ ;\ 5\ ;\ \ldots\ldots$ $S_n=\frac{n}{2}[2(1)+(n-1)(2)]$ $=n^2$ $250<n^2$ $n>\sqrt{250}$ $\therefore n>15{,}8$ The sum of the 16 first differences will be greater than 250. Therefore the 17th term of the quadratic number pattern is the first satisfying this condition./*Die som van 16 eerste verskille sal groter as 250 wees. Gevolglik sal die 17de term van die kwadratiese getalpatroon die eerste wees wat aan die voorwaarde voldoen.*	✓ $S_n=n^2$ ✓ $S_n>250$ ✓ $n>15{,}8$ ✓ $n=17$ (4)
3.2.1	$21+21(0{,}85)+21(0{,}85)^2+\ldots\ldots$ $T_n=ar^{n-1}$ $T_{10}=(21)(0{,}85)^9$ $=4{,}86$ cm	✓ $n=10\ ;\ r=0{,}85$ or $\frac{17}{20}$ ✓ substitution into correct formula ✓ answer (3)

3.2.2	$S_n=\frac{a(1-r^n)}{1-r}$ $S_{15}=\frac{21(1-(0{,}85)^{15})}{1-0{,}85}$ $=127{,}77$ Area of the page = 30 x 21 = 630 Percentage of paper covered in grey ink: $=\frac{127{,}77}{630}\times 100\%$ $=20{,}28\%$	✓ $n=15$ ✓ 127,77 ✓ 630 ✓ 20,28 (4) **[15]**